Local Edge-Preserving Multi-scale Decomposition for High Dynamic Range Image Tone Mapping

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Abstract

Proposed method
- Structuring novel filter
  - Edge-preserving decomposition of image
    - Inclusion with local means all location in Filtered image
    - Preserving salient edges

- Multi-scale decomposition
  - Application to high dynamic range image
  - Assumptions in proposed method
    - Holding local means all locations in base layer
    - Salient edges of every scale
      - large gradients in corresponding local window
    - All of nonzero gradient information belonging to detail layer
Introduction

- HDR image processing
  - Definition of dynamic range
    - Ratio of maximum to minimum intensities of scene
  - Acquisition of HDR imaging
    - Fusion of multi-exposure images
  - Compressing intensity distribution of HDR image
    - Exceeding dynamic ranges of displays
    - Compression for low-frequency on HVS
      - Less sensitive to low-frequency than high-frequency components
Tone mapping by using feature of HVS

Methods based on retinex theory

- Decomposing image by using various filters
  - Illumination image (base layer)
  - Reflectance image (detail layer)
  - Edge-preserving for avoiding halo artifacts
    » Preserving locally edge in base layer

Improvement

- Small gradient also significant for edge locally
- Holding locally salient but small gradient in base layer
Problem Statement

- Previous work
  - Retinex Theory
    • Decomposition of HDR based on Retinex Theory
      - Replacing filter in Retinex
      - Giving more satisfactory resultings
    • Image consisting of illumination \( L \) and reflectance \( R \)
      \[
      \log(I) = \log(L) + \log(R)
      \]  
      \( (1) \)
    • Decomposition of image with filter
    • Expression of R.Kimmel’s filter
      - Obtaining base layer of input image
        » Minimizing polynomial below
        \[
        \iint \left( |\nabla L|^2 + \alpha (L - I)^2 + \beta |\nabla (L - I)|^2 \right) dx dy
        \]
        \( subject \ to : L \geq I \)
        \[
        \]  
        \( (2) \)
        where \( \alpha \) , \( \beta \) is free weighting parameters.
• Edge-preserving filter for preserving image detail
  - Image consisting of base layer $B$ plus detail layer $D$
    
    $I = B + D$  

  - Expression of Z.Farbman’s filter
    » Obtaining base layer of input image
      » Minimizing polynomial below

    
    \[
    \iint \left( (B - I)^2 + \lambda \left( \alpha_x (I) \left( \frac{\partial B}{\partial x} \right)^2 + \alpha_y (I) \left( \frac{\partial B}{\partial y} \right)^2 \right) \right) dxdy
    \]

    where $I$ is luminance of input image, and $B$ is base layer,
    $\alpha_x$ and $\alpha_y$ are image information dependent coefficients,
    $\lambda$ is free weighting parameter.
- Expression of G.Guarnieri’s filter
  » Obtaining base layer of input image
    » Minimizing polynomial below

$$\iint (\omega |\nabla L|^2 + (L - I)^2) dxdy$$

subject to: $$L \geq I$$

(5)

where $$\omega$$ is space-varying coefficient.
Problem Statement

- Multi-scale decomposition image
  - Decomposing image to base layer and multiple detail layers
    \[ I = B_0 + D_1 + D_2 + \ldots + D_n \] (6)
  - Processing at every step
    - Obtaining different base layers for each step
    - Dynamic range compression and contrast enhancement
      » At every detail layer

- Assumptions of proposed method
  - Using assumptions for designing filter
    - Base layer preserving local means in all local regions
    - Salient edges of every scale
      » Treating as relatively large gradients in local window
  - Using assumption for multi-scale decomposition
    - All nonzero gradient information belonging to detail layer
Proposed Algorithm

- Showing assumptions as functions
  - Detail layer oscillating around zero
    - Assumption of base layer preserves local means
    - Representation by minimizing polynomial below
      \[ \iint_{\omega} (I - B)^2 \, dx \, dy \]
      where \( \omega \) stands for local window,
      \( I \) stands for image’s luminance,
      \( B \) stands for base layer.
    - Base layer smooth enough in local window
      - Every pixel holding constant value
        » Derived function equal to zero at every point
        \[ 2 \cdot \iint_{\omega} (I - B) \, dx \, dy = 0 \Rightarrow B = \frac{1}{N} \iint_{\omega} I \, dx \, dy \]
        where \( N \) is number of points in window \( \omega \).
– Preserving local salient edges in base layer
  • Representation by polynomial below

\[ \iint_{\omega} \left[ (I - B)^2 + \frac{\alpha}{|\nabla I|^\beta} |\nabla B|^2 \right] dxdy \]  \hspace{1cm} (9)

where \( \frac{\alpha}{|\nabla I|^\beta} \) is a coefficient balancing between two terms,
\( \alpha \) determines coefficient’s sensitivity to gradient of \( I \),
\( \beta \) is free parameter,
\( \nabla I \) stands for gradient of \( I \).
– Adding two constraints to same energy function
  
  • Obtaining local energy function

\[
\iint_\omega (I - B)^2 \, dx \, dy + \lambda \iint_\omega [ (I - B)^2 + \frac{\alpha}{|\nabla I|^\beta} |\nabla B|^2 ] \, dx \, dy
\]

\[
\Rightarrow \iint_\omega \left( (I - B)^2 + \frac{\alpha'}{|\nabla I|^\beta} |\nabla B|^2 \right) dx \, dy \tag{10}
\]

where \( \lambda \) treats balance between two constraints, and it is absorbed in \( \alpha' \).
Local Edge-Preserving Filter

- Rewriting (10) as discrete form

\[
\sum_{i \in \omega} (I_i - B_i)^2 + \frac{\alpha'}{|\nabla I_i|^\beta} |\nabla B_i|^2
\]

(11)

- Procedure of simple solution
  - Supposing \(B\) carrying linear dependence with \(I\) in local window
    - Pixels highly correlating locally \(B\)
      \[
      B_i = a_{\omega} I_i + b_{\omega}, \quad i \in \omega
      \]
      (12)
      where \(a_{\omega}\) and \(b_{\omega}\) are constant coefficients in window \(\omega\).
  - Replacing \(B\) in (11) by (12) get

\[
\sum_{i \in \omega} (I_i - a_{\omega} I_i - b_{\omega})^2 + \alpha' |\nabla I_i|^{2-\beta} \cdot a_{\omega}^2
\]

(13)
Solution of Linear least squares

\[
\begin{align*}
a_\omega &= \frac{\sigma^2}{\sigma^2_\omega + \frac{1}{N} \alpha \sum_{i \in \omega} |\nabla I_i|^{2-\beta}} \\
b_\omega &= \bar{I}_\omega - a_\omega \bar{I}_\omega 
\end{align*}
\]  

(14)

where \( \sigma^2_\omega \) is variance of \( I \) in window \( \omega \),
\( \bar{I}_\omega \) is mean of \( I \) in window \( \omega \).

Getting LEP output

\[
B_i = \frac{1}{N} \sum_{k \in \omega} (a_k I_i + b_k) = \bar{a}_i I_i + \bar{b}_i, \quad i \in \Omega 
\]  

(15)

where \( \Omega \) represents area of image,
\( \bar{a}_i \) is average of \( a_k \) in neighborhood window, and same with \( \bar{b}_i \).
- Comparison between LEP and three other filters
  - Input image with black scan line

Fig. 1.(a) Input image created by Farbman et al., with a black scan line.
• Resulting of BLF (bilateral line filter)

Fig. 1.(b) Result of the BLF.

Fig. 1.(c) Result Plot of the black scan line in (a).
• Resulting of guided filter

**Fig. 1.(d)** Result of guided filter.

**Fig. 1.(e)** Result Plot of the black scan line in (a).
- Resulting of WLS filter

**Fig. 1.(f)** Result of WLS filter.

**Fig. 1.(f)** Result Plot of the black scan line in (a).
• Resulting of LEP filter

**Fig. 1.(h)** Result of our LEP filter.  

**Fig. 1.(i)** Result Plot of the black scan line in (a).
Analysis of LEP

- Rewriting $a_\omega$ in (14)
  - Setting $a' = \beta = 1$
  - Considering one-dimensional case

\[
a_\omega = \frac{1}{1 + \frac{1}{N} \sum_{i \in \omega} \nabla I_i} = \frac{1}{1 + \frac{1}{\sigma_\omega^2} \sum_{i \in \omega} |I_i - I_{i+1}|} \quad \frac{1}{1 + \frac{1}{\sum_{i \in \omega} (I_i - \bar{I})^2}}
\]

- Making $R = \frac{\sum_{i \in \omega} |I_i - I_{i+1}|}{\sum_{i \in \omega} (I_i - \bar{I})^2}$, then, $a_\omega = \frac{1}{1 + R}$
Analysis of parameter $R$

- Getting expression of $R$

$$R = \frac{\sum_{i \in \omega} |I_i - I_{i+1}|}{\sum_{i \in \omega} (I_i - \bar{T})^2} = \frac{\sum_{i \in \omega} |\delta_i - \delta_{i+1}|}{\sum_{i \in \omega} \delta_i^2}$$ \hspace{1cm} (17)

where $\delta_i$ denotes deviation from luminance $I_i$ to luminance mean $\bar{T}$.

- Fixing denominator
  - Same absolute deviation of every $\delta_i$
• Analysis of typical signal
  - Edge instance
    » Considering one case for (17)

\[
R_a = \frac{\sum_{i \in \omega} |\delta_i - \delta_{i+1}|}{\sum_{i \in \omega} \delta_i^2} = \frac{|\delta_5 + \delta_6|}{M}
\]

(18)

where $M$ denotes fixed value for denominator, $\delta_5, \delta_6$ are shown in Fig.2.(a).

» Showing example of typical signal figure

![Smooth signal except for a salient edge.](a)

**Fig. 2.(a)** Smooth signal except for a salient edge.
Another edge instance

» Considering another case for (17)

\[ R_b = \frac{\sum_{i=0}^{\omega} |\delta_i - \delta_{i+1}|}{\sum_{i=0}^{\omega} \delta_i^2} = \frac{2 \cdot \sum |\delta_i|}{M} \]  

(19)

» Showing example of typical signal figure

![Oscillating signal figure](image)

Fig. 2.(b) Oscillating signal.
- Conclusion of analysis
  » Considering $R_b > R_a$
    » Compressing signal in (b) heavier
    » Preserving salient edge in (a)
  » Preserving local edge
    » Similar to case of Fig.2.(a)
Parameter for LEP

- Giving two parameters
  - Determining sensitivity of filter to gradient
  - Treating more gradients as local salient edge
    - $\alpha'$ and $\beta$ with small value
  - Selecting satisfactory value
    - $\alpha' = 0.1$ and $\beta = 1$
Filtered resulting of LEP with varying parameters

**Fig. 3.** Filtered results of LEP with varying parameter values. Original image is shown at top with multi-scale noise representing image pattern or details. The central image presents a satisfactory.
Fig. 3. Comparison of progressively coarsening effect between our algorithm and other two algorithms. Plot is shown with first base layer (red lines) and second base layer (pink lines). (a) Iterative filtering using WLS. (b) Coarsen using method in [10]. (c) Iterative filtering using our LEP with blue line representing mean of pick line.
Multi-scale Decomposition

- Decomposing base layer progressively

\[ B_{l-1} = LEP_l(B_l), \text{ for } l = n, \ldots, 2, \text{ and } B_n = I \]  \hspace{1cm} (20)

where \( LEP_l \) denotes filter function, \( l \) is scale level.

- Getting last base layer as \( B_0 = \text{mean}(B_1) \)
- Obtaining detail layer simultaneously

\[ D_l = B_l - B_{l-1}, \text{ for } l = n, \ldots, 2. \]  \hspace{1cm} (21)

- Decomposing image into three detail and one base layer

\[ I = B_0 + D_1 + D_2 + D_3 \]  \hspace{1cm} (22)
Dynamic Range Compression

- Calculating compression function

\[ y = 2 \cdot \frac{\arctan(x \cdot 20)}{\pi} \]  

(23)

- Plot of compression function

Fig. 5. Plot of our compression function. It is convex and S shaped.
Color

- Using mapping function for three color channels

\[
C_{\text{out}} = \left( \frac{C_{\text{in}}}{L_{\text{in}}} \right)^s \cdot L_{\text{out}}
\]    

(24)

where \( C = R, G, B \) represents three color channels,
\( L_{\text{in}}, L_{\text{out}} \) denote luminance before and after HDR compression.
\( s \) default value is 0.6.
Experimental Results and Discussion

- Implementation
  - Transforming HDR radiance map into gray image ranging
    - Typical operation of most methods
      - Averaging three channels for getting luminance
      - Transforming luminance into logarithm domain
        \[ L = \ln(L_{in} \cdot 10^6 + 1) \] 
        (25)
    - Normalized operation
      \[ \tilde{L} = L / \max(L) \] 
      (26)
  - Decomposing image
    - Setting values for \( \alpha' = 0.1, \beta = 1 \)
    - Using local windows
      - radius \( r = 2 \) for first decomposition
      - radius \( r = 20 \) for second decomposition
• Resulting images of different choose of radiuses

Fig. 6. Results of various combinations of radiuses. Image courtesy of Fredo Durand. (a) Result with first window radius $r = 2$, second $r = 20$. (b) Result with first $r = 20$ and second $r = 100$. (c) Result with first $r = 2$, and second $r = 200$. (d) Result with first $r = 20$ and second $r = 200$. 
– Composing image
  • Image composing by detail layers
    \[ L_{out} = D'_1 \cdot 0.5 + D'_2 + D'_3 \]  
  • Cutting low and high values
    – Getting stretched histogram
      » Reducing noise artifact
      » Increasing major contrast
• Demonstration of cut and stretch effect

![Image courtesy of Karol Myszlowski. (a) Image without the cut and stretch. (b) Image looks clearer with the cut and stretch.](image)

**Fig. 7.** Results Demonstration of the cut and stretch effect. Image courtesy of Karol Myszlowski. (a) Image without the cut and stretch. (b) Image looks clearer with the cut and stretch.
Fig. 7. Results Demonstration of the cut and stretch effect. Image courtesy of Karol Myszlowski. (c) Histogram of (a). There are few pixel at the high and low ends. (d) Stretched histogram of (b).
Fig. 8. Diagram of our algorithm.
- Results and discussion
  - Comparing HDR reproduce with other three algorithms

Fig. 9. Comparison of real HDR reproducing image between our algorithm and other three algorithms. The close-ups show images in rectangles, respectively. (a) Result of [15] using BLF. (b) Result of [9] using WLS.
Other pairs of figure 9

**Fig. 9.** Comparison of real HDR reproducing image between out algorithm and other three algorithms. The close-ups show images in rectangles, respectively. (c) Result of [10]. (d) Our result.
• Other pairs of figure 9

Fig. 9. Comparison of real HDR reproducing image between out algorithm and other three algorithms. The close-ups show images in rectangles, respectively. (e) Close-up of(a). (f) Close-up of(b). (g) Close-up of(c). (h) Close-up of(d).
• Quantitative measure for Fig.9
  - Defining measure as normalized sum of total gradients

\[ S = \frac{1}{N} \sum |\nabla I| \]

where \( N \) is number of pixels in image \( I \).

Table 1. Quantitative Measure for Fig.9.

<table>
<thead>
<tr>
<th>Image</th>
<th>Sharpness</th>
<th>Naturalness</th>
<th>Structural fidelity</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 9(a) BLF [15]</td>
<td>6.2344</td>
<td>0.9800</td>
<td>0.9045</td>
<td>0.9655</td>
</tr>
<tr>
<td>Fig. 9(b) WLS [9]</td>
<td>6.0238</td>
<td>0.9932</td>
<td>0.9269</td>
<td>0.9751</td>
</tr>
<tr>
<td>Fig. 9(c) local extrema [10]</td>
<td>9.5101</td>
<td>0.7957</td>
<td>0.9188</td>
<td>0.9404</td>
</tr>
<tr>
<td>Fig. 9(d) LEP</td>
<td>10.9310</td>
<td>0.9993</td>
<td>0.9285</td>
<td>0.9766</td>
</tr>
</tbody>
</table>

The best ones are shown in bold.
Comparison reproduced HDR images

- WLS filter and LEP filter

**Fig. 10.** Comparison of reproduced HDR images obtained by same process but using different filters. (a) WLS filter. (b) LEP filter.
Fig. 10. Comparison of reproduced HDR images obtained by same process but using different filters. (c) Close-ups of red rectangle areas in (a). (d) Close-ups of red rectangle areas in (b).
• Quantitative measure for Fig. 10

**Table 2.** Quantitative Measure for Fig. 10.

<table>
<thead>
<tr>
<th>Image</th>
<th>Sharpness</th>
<th>Naturalness</th>
<th>Structural fidelity</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 10(a) WLS</td>
<td>11.0498</td>
<td><strong>0.7464</strong></td>
<td>0.9400</td>
<td>0.9392</td>
</tr>
<tr>
<td>Fig. 10(b) LEP</td>
<td><strong>16.0674</strong></td>
<td>0.7339</td>
<td><strong>0.9489</strong></td>
<td><strong>0.9400</strong></td>
</tr>
</tbody>
</table>

The best ones are shown in bold.
Comparison of reproduced memorial church HDR images

- Proposed algorithm and other seven algorithms

**Fig. 11.** Comparison of reproduced famous memorial church HDR image between our algorithm and other seven algorithms. (a) Our result. (b) Result taken directly from [14].
Fig. 11. Comparison of reproduced famous memorial church HDR image between our algorithm and other seven algorithms. (c) Result taken directly form [15]. (d) Result form [16] after tweaking parameters.
Fig. 11. Comparison of reproduced famous memorial church HDR image between our algorithm and other seven algorithms. (e) Result taken directly from [3]. (f) Result taken directly from [17].
Fig. 11. Comparison of reproduced famous memorial church HDR image between our algorithm and other seven algorithms. (g) Result taken directly from [19]. (h) Result taken directly from [20].
Conclusion

- Proposed method
  - Presenting three assumptions for image decomposition
    - Deriving local edge-preserving filter
  - Comparing with recent effective algorithms
    - Preserving local tiny details
    - Appealing global view

- Experimental results
  - LEP filter giving more pleasing view than pervious methods
    - Avoiding halo effect
    - Emphasis on local edge preservation