Recovery of Spectral Reflectance of Objects Being Imaged Without Prior Knowledge

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Abstract

- Important factor of color image acquisition
  - Prior knowledge of noise
    - Estimating colorimetric values
    - Recovering spectral reflectances of pixels of objects being imaged

- Proposed method
  - Determining noise variance of multispectral color image acquisition system
  - Recovering spectral reflectances of art painting
    - Use of sensor responses without prior knowledge of objects being imaged and noise
Introduction

- Main aim of color image acquisition device
  - Acquisition of accurate colorimetric information
  - Recovery of spectral reflectance of pixel of objects being imaged
Estimation of colorimetric values using sensor responses
- Considering Factors about accuracy of estimates
  - Spectral sensitivities about set of sensors
  - Noise

Reconstruction of reflectances using sensor responses
- Considering factors about accuracy of estimates
  - Spectral sensitivities about set of sensors
  - Noise
  - Recovery of spectral reflectance of object being imaged
    - Application of Wiener filter
      - Using prior knowledge of noise variance of image acquisition system
        » Including all sensor response error
Proposed method

- Recovering spectral reflectances about pixels of image
  - Not using prior knowledge
    - Spectral reflectances
    - Noise
Model

- Proposed model
  - Vector notation for color reproduction
    - Definition of sensor response
      \[ p = SLr + e \] (1)

where \( p \) is a \( M \times 1 \) sensor response vector from the \( M \) channel sensors, \( S \) is a \( M \times N \) matrix of the spectral sensitivities of sensors, \( L \) is a \( N \times N \) diagonal matrix with samples of the spectral power distribution of an illuminant along the diagonal, \( r \) is a \( N \times 1 \) spectral reflectance vector, and \( e \) is a \( M \times 1 \) additive system noise vector.
Expression of recovered surface reflectance

- Assumption
  - Uncorrelation between \( r \) and \( e \)
  - Noise mean: 0

\[
\hat{r} = R_{ss}S_L^T \left( S_L R_{ss}S_L^T + R_{ee} \right)^{-1} p
\]

where \( \hat{r} \) is a covered surface reflectance vector, \( T \) represents the transpose of a matrix, \( R_{ss} \) is an autocorrelation matrix of the spectral reflectances of samples which will be captured by a device, and \( R_{ee} \) is also an autocorrelation matrix of the noise used for the Wiener estimation.
– Estimation error vector $\Delta \mathbf{r}$ between actual and recovered vector for surface reflectance vector $\mathbf{r}$

• Assuming noise as uncorrelated random variable
  – Substituting $\mathbf{R}_{ee} = \sigma_e^2 \mathbf{I}$

$$\Delta \mathbf{r} = \mathbf{r} - \mathbf{R}_{ss} \mathbf{S}_L^T \left( \mathbf{S}_L \mathbf{R}_{ss} \mathbf{S}_L^T + \sigma_e^2 \mathbf{I} \right)^{-1} \mathbf{p}$$  \hspace{1cm} (3)

• Representing auto correlation matrix $\mathbf{R}_{ss}$
  – Using set of eigenvectors and eigenvalues of matrix
    » Substitution of $\mathbf{R}_{ss} = \mathbf{V} \Lambda \mathbf{V}^T$

$$\mathbf{S}_L^V = \mathbf{S}_L \mathbf{V} \Lambda^{1/2}$$  \hspace{1cm} (4)

where $\mathbf{V}$ represents the basis matrix, $\Lambda$ is $N \times N$ a diagonal matrix with positive eigenvalues of the matrix along the diagonal in decreasing order.
• Modification of $\Delta r$
  - Substitution of $R_{ss} = V \Lambda V^T$
  - Substitution of Eq.(4) into Eq.(3)

$$\Delta r = r - V \Lambda^{1/2} S_L^{V^T} \left( S_L^{V^T} S_L^{V^T} + \sigma_e^2 I \right)^{-1} p$$ (5)

» Singular value decomposition of matrix $S_L^{V}$

$$S_L^{V} = \sum_{i=1}^{\beta} \kappa_i^{y} d_i^{y} b_i^{y^T}$$ (5)

where $\beta = \text{Rank}(S_L^{V})$, $\kappa_i^{y}$, $d_i^{y}$, and $b_i^{y}$ represent in ith singular value, the ith left and right singular vector, respectively.
Substitution of Eq. (6) into Eq. (5)

\[
S_L^V \left( S_L^V S_L^V + \sigma_e^2 \right)^{-1} = \sum_{i=1}^{\beta} \frac{k_i^V}{k_i^V + \sigma_e^2} b_i^V d_i^V^T
\]  

(6)

\[
\Delta r = r - VA^{1/2} \sum_{i=1}^{\beta} \frac{k_i^V}{k_i^V + \sigma_e^2} b_i^V d_i^V^T (S_L r + e)
\]  

(7)
– Representation of mean square error (MSE)

- Averaging square of Euclid norm for error vector $||\Delta r||^2$ over surface reflectance spectra

$$\text{MSE} \left( \sigma_e^2, \sigma^2 \right) = \mathbb{E} \left\{ \|r^2\| \right\} = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N} \sum_{j=1}^{\beta} \lambda_i b_{ij}^2 + \sum_{i=1}^{N} \sum_{j=1}^{\beta} \frac{\sigma_e^4 + K_j v^2 \sigma^2}{K_j^v + \sigma_e^2} \lambda_i b_{ij}^2$$  \hspace{1cm} (8)

where $b_{ij}$ and $\lambda_i$ represent the $i$th row of the right singular vector $b_j^v$, $j = 1, \ldots, \beta$ of $S_L^v$ and $i$th eigenvalue of the $R_{ss}$, respectively.

- Representation of $\text{MSE}_{\text{min}}$

$$\text{MSE}_{\text{min}} = \text{MSE} \left( \sigma^2, \sigma^2 \right) = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N} \sum_{j=1}^{\beta} \lambda_i b_{ij}^2 + \sum_{i=1}^{N} \sum_{j=1}^{\beta} \frac{\sigma^2}{K_j v^2 + \sigma^2} \lambda_i b_{ij}^2$$ \hspace{1cm} (9)
• Representation of MSE
  - Substitution of $\sigma_e^2$

$$\text{MSE}(0, \sigma^2) = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N} \sum_{j=1}^{\beta} \lambda_i b_{ij}^2 + \sum_{i=1}^{N} \sum_{j=1}^{\beta} \frac{\sigma^2}{\kappa_j^2} \lambda_i b_{ij}^2$$ (10)

• Representation of $\text{MSE}_{\text{free}}$

$$\text{MSE}_{\text{free}} = \text{MSE}(0, 0) = \sum_{i=1}^{N} \lambda_i - \sum_{i=1}^{N} \sum_{j=1}^{\beta} \lambda_i b_{ij}^2$$ (11)

  - Representation of estimated system noise variance

$$\hat{\sigma}^2 = \frac{\text{MSE}(0, \sigma^2) - \text{MSE}_{\text{free}}}{\sum_{i=1}^{N} \sum_{j=1}^{\beta} \frac{\lambda_i b_{ij}^2}{\kappa_j^2}}$$ (12)
Direct approaches for noise variance estimation

- First expression
  - Using Eq.(1)

\[
\hat{\sigma}^2 = M^{-1} \text{Tr} \left( E \{ee^T\} \right) = M^{-1} E \left\{ \|p - S_L r\|^2 \right\}
\]  

(11)

where M represents the number of sensors.

- Second expression

\[
\hat{\sigma}^2 = M^{-1} E \left\{ \text{Tr} \left( pp^T \right) - \text{Tr} \left( (S_L r)(S_L r)^T \right) \right\}
\]  

(11)
Experimental procedures and results

- Experimental procedures
  - Using two sets of spectral sensitivities
    - Acquisition of two color charts

Fig. 1. Spectral sensitivities of two sets of sensors used in the experiments.
– Using halogen lamp for image capture

Fig. 2. Spectral power distributions of a Halogen lamp.
Experimental results of noise variance estimates
- Expressing recovered reflectance spectra of two color charts

**Fig. 3.** Typical examples of the recovered reflectance spectra of the Kodak Q60R1 and the Macbeth ColorChecker using the noise variance estimated by the proposed procedures.
Table 1. Estimated parameters for Kodak Q60R1 by multispectral cameras

<table>
<thead>
<tr>
<th>Sensors</th>
<th>MSE(0,σ²)</th>
<th>MSE(σ²opt,σ²)</th>
<th>MSE(σ²,σ²)</th>
<th>MSE(σ²exp,σ²)</th>
<th>MSE(σ²exc,σ²)</th>
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<th>σ²</th>
<th>δ²</th>
<th>δ²exp</th>
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### Table 2. Estimated parameters Macbeth ColorChecker by multispectral cameras

<table>
<thead>
<tr>
<th>Sensors</th>
<th>MSE(0, σ²)</th>
<th>MSE(σ², σ²)</th>
<th>MSE(σ²opt, σ²)</th>
<th>MSE(σ²rep, σ²)</th>
<th>MSE(σ²rec, σ²)</th>
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Table 3. Estimates of the MSE using the system noise variance which is estimated by another spectral reflectances

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Kodak Q60R1</th>
<th>Macbeth ColorChecker</th>
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<tbody>
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<td>MSE($\sigma_{opt}^2, \sigma^2$)</td>
<td>MSE($\sigma_{opt}^2, \sigma^2$)</td>
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<td>0.018786</td>
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Experimental results of recovery of spectral reflectances of art painting without prior knowledge of object being imaged

- Test image of art painting

Fig. 4. Art painting used in the experiment.
Expressing recovered reflectance spectra of art painting

Fig. 5. Typical examples of the recovered spectral reflectances: (a)-(c) points of art painting.
Table 4. Summary of the estimated parameters by the proposal

<table>
<thead>
<tr>
<th>Rss</th>
<th>MSE(0, $\sigma^2$)</th>
<th>MSE($\hat{\sigma}^2$, $\sigma^2$)</th>
<th>MSE($\hat{\sigma}_{opt}^2$, $\sigma^2$)</th>
<th>$\Delta E_{ab}^*$</th>
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<tr>
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<td>Munsell Middle</td>
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<td>Munsell Low</td>
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Conclusion

- **Proposed method**
  - **Purpose**
    - Recovering surface reflectance spectra of objects being imaged
      - Use of sensor responses
        - Not using reflectance spectra of object
        - Not using noise
  - **Procedure**
    - Estimating system noise variance of color image acquisition system
      - Reconstructing of reflectance spectra of pixels of objects being imaged
        - Use of Wiener filter