Video Super-resolution Reconstruction Based on Sub-pixel Registration and Iterative Back Projection

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Abstract

◆ Proposed method

– Improvement of spatial resolution of video
  • Use of sliding window in registration
  • Registration algorithm
    – Use of Four-parameter transformation model through Taylor series expansion
    – Iterative solving method
    – Gaussian pyramid image model
  • Reconstruction algorithm
    – Use of iterative back projection algorithm
Introduction

◆ Necessity of super-resolution
  – Limit of camera resolution
    • Spatial limit
      – Determining by spatial density of optical sensor
        » Limitation of smallest spatial size of observed object in video
    • Temporal limit
      – Determining by frame rate and the exposure time
- Solving limitation of camera resolution
  
  • Direct method
    - Improving imaging system by manufacturing technique
      » Pixel density of CCD
      » Lens size
  
  • Proposed method
    - Use of Super-resolution reconstruction
      » Use of spatial sub-pixel movement information between frame
      » Reconstruction from a low-spatial-resolution video to high-spatial-resolution video
Categories of registration algorithm

- Frequency domain method
  - Use of two shifted image in frequency domain
    - Different by phase shift
  - Applying log-polar transformation of magnitude of frequency spectra
    - Conversion from image rotation and scale to horizontal and vertical shift

- Spatial domain method
  - Consideration of more general motion model
  - Using whole image or on set of selected corresponding feature vector
Frequency domain methods

- Lucchese and Cartelazzo
  - Development of rotation estimation algorithm
  - Estimating horizontal and vertical shift by using phase correlation method

- Vandewalle et al
  - Proposal of planar motion estimation method
  - Registration of aliased low resolution images through Fourier transformation
  - Getting movement parameter with sub-pixel precision for image
◆ Spatial domain methods
  – Keren et al
    • Development of iterative planar motion estimation algorithm based on Taylor expansions
    • Getting movement parameter with high sub-pixel precision in cases of small angle
  – Bruce and Kanade
    • Use of spatial intensity gradient of images
    • Finding good match through Newton-Raphson interaction
  – Wu et al
    • Use of coarse-to-fine wavelet-based motion model
    • Estimation of dense motion vector between two image
Reconstruction

- Use of estimated movement information in registration
  - Tsai and Huang
    - Using Fourier coefficients of the high-resolution image as function of Fourier coefficient of registered low-resolution image
  - Vandewalle et al
    - Use of interpolation based method
    - Creating high-resolution image from multiple low-resolution image
• Pham et al
  – Proposal of adaptive normalized convolution method
  – Fusing irregularly sampled data
  – Using projection onto convex sets (POCS) algorithm, posteriori (MAP) statistical method, and iterative back-projection (IBP) method

• Eld and Feuer
  – Presentation of super-resolution frame-work
  – Combining maximum likelihood/MAP approach with POCS approach
Advantages of the proposed method

- Improvement in registration
  - Precision of registration
  - Calculation speed
- Considering boundary frames in reconstruction
Video Super-resolution Reconstruction Model

- Super-resolution reconstruction model
  - Assuming existence of temporal relativity between the adjacent frame
  - Use of adjacent frames for reconstruction

![Diagram of super-resolution reconstruction model of a video.](image-url)
Movement Registration Method

- **Registration algorithm**
  - Consideration of the case of planar movement
  - Denoting movement between reference image $r(x', y')$ and registered image $g(x, y)$ by three parameters
    - Horizontal shift ($\Delta x$)
    - Vertical shift ($\Delta y$)
    - Rotation angle ($\theta$)
  - Rigid transformation model between the coordinate of two image
    
    $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$  
    
    (1)
• Avoiding hypothesis of small angle in Taylor series expansion

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} 1 + a_1 & -a_2 \\ a_2 & 1 + a_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}
\]  

where \( a_1, a_2, a_3, \) and \( a_4 \) are four parameters of transformation model.

• Mathematical relationship between reference image and registered image

\[
g(x, y) = r(x + a_1 x - a_2 y + a_3, y + a_2 x + a_1 y + a_4)
\]

(3)

• Two-dimensional series expansion and ignoring high order term

\[
g(x, y) \approx r(x, y) + (a_1 x - a_2 y + a_3) \frac{\partial r}{\partial x} + (a_2 x + a_1 y + a_4) \frac{\partial r}{\partial y}
\]

(4)
• Object function

\[ E(a_1, a_2, a_3, a_4) = \sum \left[ r(x, y) + (a_1x - a_2y + a_3) \frac{\partial r}{\partial x} + (a_2x + a_1y + a_4) \frac{\partial r}{\partial y} - g(x, y) \right]^2 \]

(5)

where \( \sum \) represent the summation to overlapped part of \( r \) and \( g \).

• Getting optimal registered parameter

\[
\left[ \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4 \right] = \arg \min_{(a_1, a_2, a_3, a_4)} E(a_1, a_2, a_3, a_4)
\]
• Performing partial derivative about \( a_1 \), \( a_2 \), \( a_3 \), and \( a_4 \) to Eq. (5)

\[
X = C^{-1}V
\]  

(7)

where

\[
X = \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \end{bmatrix}, \quad V = \begin{bmatrix} \sum D_1 (g-r) \\ \sum D_1 (g-r) \\ \sum \frac{\partial r}{\partial x} (g-r) \\ \sum \frac{\partial r}{\partial y} (g-r) \end{bmatrix}, \quad C = \begin{bmatrix} \sum D_1 D_2 & \sum D_1 D_3 & \sum D_1 \frac{\partial r}{\partial x} & \sum D_1 \frac{\partial r}{\partial y} \\ \sum D_2 D_1 & \sum D_2 D_3 & \sum D_2 \frac{\partial r}{\partial x} & \sum D_2 \frac{\partial r}{\partial y} \\ \sum \frac{\partial r}{\partial x} & \sum \frac{\partial r}{\partial y} & \sum \frac{\partial r}{\partial x} \frac{\partial r}{\partial y} & \sum \frac{\partial r}{\partial y} \frac{\partial r}{\partial y} \\ \sum \frac{\partial r}{\partial y} & \sum \frac{\partial r}{\partial x} & \sum \frac{\partial r}{\partial y} \frac{\partial r}{\partial x} & \sum \frac{\partial r}{\partial y} \frac{\partial r}{\partial y} \end{bmatrix}
\]

\[
D_1 = x \frac{\partial r}{\partial y} - y \frac{\partial r}{\partial x}, \text{ and } D_2 = x \frac{\partial r}{\partial x} + y \frac{\partial r}{\partial y}
\]

• Transforming optimal registered parameter by change from four-parameter model to three-parameter transformation

\[
\Delta x = \hat{a}_3, \quad \Delta y = \hat{a}_4, \quad \theta = \arcsin(\hat{a}_2) (180/\pi)
\]  

(8)
– Iterative solution method

• Improvement of precision
• Expanding applicable area of registration algorithm
• Estimation of optimal parameters

– Use of following iterative approximation method

\[ X_{k+1} = C_k^{-1} V_k + X_k \]  \hspace{1cm} (9)

Where \( k \) is the iteration number,

\( X_{k+1} \) and \( X_k \) denote registered parameter gained in

\((k+1)\)' th and \( k\)' th iteration.
- Obtaining process of optimal parameter
  1) Shifting and rotating reference image \( r \) by \( X_k \)
  2) Denoting new image \( r' \) by 1)
  3) Registering image \( r' \) with image \( g \)
  4) Obtaining current registered parameter \( C_k^{-1}V_k \)
  5) Repeating process by substituting \( X_k \) with \( X_{k+1} \)
- Gaussian pyramid image model
  - Use of three-level Gaussian pyramid image model
    - Improvement of computing speed
    - Enhancing robustness to noise of registration algorithm
  - In each level, estimating optimal movement parameter through iterative approximation method
    - Obtaining process of optimal parameter
      1) Obtaining movement registration from third-level image with the coarsest resolution
      2) Warping reference image in second level by using estimation of movement parameter in third level
      3) Warping reference image in first level by using summation of the estimated optimal movement parameter in third level and in second level
Super-resolution Reconstruction Algorithm

- Iterative back projection method
  - Regarding each practical low-resolution image as projection of the real scene

Fig. 2. Sketch map of the IBP method.
Mathematical description of the iterative back projection algorithm

\[
\hat{f}^{k+1} = \hat{f}^k - \lambda \sum_{i=1}^{P} H_{i}^{BP} (\hat{y}_{i}^{k} - y_{i})
\]

(10)

where \( k \) is the iteration number,
\( \hat{f}^{k+1} \) and \( \hat{f}^k \) are super-resolution image gained in \((k+1)\)' th and \( k \)' th iteration,
\( \hat{y}_{i} \) is i’th low-resolution image of \( \hat{f}^k \) under low-resolution image observation model,
\( \lambda \) is gradient step.
Experiments

◆ Simulation experiment
  – Evaluation of proposed method
    • Comparison between previous algorithm and proposed algorithm
    • Precision of registration algorithm
      – Use of different registration algorithm
    • Performance of reconstruction algorithm
      – Use of movement information and image
– Small movement

• Adding zero-window to original image for avoiding boundary effect
• Use of low-resolution image observation model
• Generation procedure of low-resolution image
  1) Warped high-resolution image because of different horizontal shift, vertical shift, and rotation
  2) Convolving warped image with Gaussian point spread function (PSF)
  3) Down-sampling blurred image with a factor of 2
  4) Adding Gaussian white noise to down sampled image
Movements registration error of \(i\)’th image relative to reference image is:

\[
\text{err}_i = (|\Delta x_i| + |\Delta y_i| + |\Delta \theta_i|) / 3, \quad |\Delta x_i| = |\hat{x}_i - x_i|,
\]

\[
|\Delta y_i| = |\hat{y}_i - y_i|, \quad |\Delta \theta_i| = |\hat{\theta}_i - \theta_i|
\]

where \(\hat{x}_i\), \(\hat{y}_i\), and \(\hat{\theta}_i\) are estimated horizontal shift, vertical shift, and rotation angle of \(i\)’th image, \(x_i\), \(y_i\), and \(\theta_i\) are original horizontal shift, vertical shift, and rotation angle of \(i\)’th image, \(|\Delta x_i|\), \(|\Delta y_i|\), and \(|\Delta \theta_i|\) denote absolute value of horizontal error, vertical error, and rotation error registered by \(i\)’th image.
– Average of movement registration error for all image

\[
\text{ave\_err} = \frac{\sum_{i=1}^{N} err_i}{L}
\]  

(12)

where \( L \) is total number of low-resolution image.

\[= \frac{1}{24} \sum_{i=1}^{36} \text{err}_i / L \]

\[= \frac{1}{36} \sum_{i=1}^{36} \text{err}_i / 36 \]

\[= \frac{1}{720} \sum_{i=1}^{720} \text{err}_i / 720 \]

\[= \frac{1}{540} \sum_{i=1}^{540} \text{err}_i / 540 \]

Fig. 3. Movement registration error.
– Evaluating effect of registration algorithm on quality of reconstructed image

\[
PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right), \quad MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \hat{f}(i, j) - f(i, j) \right]^2
\]

where \( f \) is original high-resolution image gained by adding zero-window around “cameraman”, \( \hat{f} \) is image reconstructed by bilinear interpolation or by super-resolution algorithm.

Fig. 4. PSNR values resulting from different registration algorithms.

Fig. 5. PSNR values resulting from different reconstruction algorithms.
- Determining suitable number of frames in video super-resolution reconstruction
  
  • Use of improved signal-to-noise ratio (ISNR) for comparing quality of reconstructed image

\[
\text{ISNR} = 10 \log_{10} \left( \frac{\| f - f_0 \|^2}{\| f - \hat{f} \|^2} \right)
\]  

(14)

where \( f_0 \) is up-sampling of reference image through bilinear interpolation.

Fig. 6. ISNR values resulting from our algorithm.
– Selecting reasonable value of iteration number in the IBP algorithm

Fig. 7. Convergence curve of the IBP algorithm.
– Use of selected value

Fig. 8. Images reconstructed by (a) up-sampling by bilinear interpolation, (b) The Vanderwalle et al. method, (c) the Pham et al. method, and (d) The IBP algorithm.
– Large movement
  • Testing performance of our registration algorithm in case of large rotation angle
  – Image reconstruction thorough IBP algorithm

**Table 1.** The original movement parameters.

<table>
<thead>
<tr>
<th>Directions</th>
<th>Movement Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$ (in pixels)</td>
<td>0  -1.3634  -1.1766  2.5656  1.5423</td>
</tr>
<tr>
<td>$y_0$ (in pixels)</td>
<td>0  -1.8756   2.8473  -1.9945  1.0166</td>
</tr>
<tr>
<td>$\theta_0$ (in degrees)</td>
<td>0  -34.8387  -14.3463  13.8344  43.5445</td>
</tr>
</tbody>
</table>
Table 2. The absolution error by different registration algorithms.

<table>
<thead>
<tr>
<th></th>
<th>Lucchese and Cortesayo\textsuperscript{11}</th>
<th>Vandewalle \textit{et al.}\textsuperscript{12}</th>
<th>Keren \textit{et al.}\textsuperscript{16}</th>
<th>Our Registration Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\Delta x</td>
<td>$ (in pixels)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.6366</td>
<td>1.9875</td>
<td>0.0911</td>
<td>0.0093</td>
</tr>
<tr>
<td></td>
<td>69.6766</td>
<td>0.0308</td>
<td>0.0599</td>
<td>0.0043</td>
</tr>
<tr>
<td></td>
<td>0.5656</td>
<td>0.0166</td>
<td>0.0446</td>
<td>0.0035</td>
</tr>
<tr>
<td></td>
<td>0.5423</td>
<td>1.5946</td>
<td>0.3991</td>
<td>0.1172</td>
</tr>
<tr>
<td>$</td>
<td>\Delta y</td>
<td>$ (in pixels)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.8756</td>
<td>0.7894</td>
<td>0.1430</td>
<td>0.0572</td>
</tr>
<tr>
<td></td>
<td>65.1527</td>
<td>0.0208</td>
<td>0.0438</td>
<td>0.0070</td>
</tr>
<tr>
<td></td>
<td>0.9945</td>
<td>0.0114</td>
<td>0.0373</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>0.0166</td>
<td>0.1217</td>
<td>0.2137</td>
<td>0.0088</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \theta</td>
<td>$ (in degrees)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.5065</td>
<td>4.9387</td>
<td>0.9052</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>9.8802</td>
<td>0.1463</td>
<td>0.6130</td>
<td>0.0009</td>
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<tr>
<td></td>
<td>1.8035</td>
<td>0.1656</td>
<td>0.5547</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>0.7210</td>
<td>13.5445</td>
<td>1.4818</td>
<td>0.1090</td>
</tr>
<tr>
<td>$\text{ISNR}$ (in decibels)</td>
<td>−10.1205</td>
<td>−3.4635</td>
<td>0.6415</td>
<td>3.1319</td>
</tr>
<tr>
<td>$\text{PSNR}$ (in decibels)</td>
<td>31.1205</td>
<td>33.1538</td>
<td>34.6985</td>
<td>35.8561</td>
</tr>
</tbody>
</table>
-Reconstruction of text image
  - Comparing performance of the reconstructed algorithm on text image

Fig. 9. Text images reconstructed by (a) bilinear interpolation (PSNR=28.3780 dB), (b) the Vandewalle et al. method (PSNR=28.6372 dB), (c) the Pham et al. method (PSNR=28.5697 dB), and (d) the IBP algorithm (PSNR=30.0801 dB).
Super-resolution reconstruction of actual color video

- Converting RGB color space into YCbCr color space
  - Under premise of guaranteeing quality of reconstructed image
- Performing process only on Y component
  - Decrease of computational amount
– Small movement
  • Comparing performance of reconstruction algorithm in this paper

**Fig. 10.** Image of the 66\(^{\text{th}}\) frame reconstructed by (a) the references Frame up-sampled by bilinear interpolation, (b) the Vanderwalle et al. method, (c) the Pham et al. method, and (d) the IBP algorithm.
− Large movement
  • Comparing registration algorithm

**Fig. 11.** Image of the sixth frame reconstructed by (a) the Lucchese and Cartelazzo method, (b) the Vandewalle et al. method, (c) the keren et al. method, and (d) our registration algorithm.
Table 3. The estimated movement parameters of the sixth sliding windows using our registration algorithm.

<table>
<thead>
<tr>
<th>Directions</th>
<th>Estimated Movement Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x}$ (in pixels)</td>
<td>$-7.3330$</td>
</tr>
<tr>
<td>$\hat{y}$ (in pixels)</td>
<td>$3.6134$</td>
</tr>
<tr>
<td>$\hat{\theta}$ (in degrees)</td>
<td>$-32.6269$</td>
</tr>
</tbody>
</table>
Conclusions

Proposed method

- Super-resolution reconstruction
  - Use of spatial sub-pixel movement information between frame gained by registration algorithm
  - Performing reconstruction to IBP algorithm
  - Converting RGB color space into YCbCr color space
    - Super-resolution reconstruction of color video sequence

- Experimental result
  - Suitable length of the sliding window
  - Reasonable iteration number for the IBP algorithm
  - Comparison of previous method and proposed method