Globally Optimized Linear Windowed Tone-Mapping

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Abstract

 Introducing new tone-mapping operator
  – Use of small overlapping windows over input image
    • Application of local linear adjustment at each window
      – Preserving monotonicity of radiance values
  – Use of guidance map
    • Effectively suppressing local high contrast while preserving details
  – Preservation of image structures
    • Abrupt radiance change or relatively smooth
  – Synthesizing HDR image from LDR image
Flow chart

1. Input an HDR image with radiance map $I^h$
2. Generate guidance map
3. Construct matrix $S$ and $B$
4. Compute $I^l$ by solving linear system
5. Restore RGB channels $I_k^l$ in the tone mapped result
6. Output an ordinary image with color channels $I_k^l$
High dynamic range compression

- Use of global linear scaling
  - Linearly flattened image structures
    - Loss of visual information mostly in highlights and shadows
  - Proposal of non-linear mapping functions
    - Global tone response curve
    - Local tone reproduction
◆ Proposal of new tone reproduction operator
  – Categorized into two types in an HDR image
    • Significant HDR with smooth radiance transition
    • Sharp and significant local radiance change
  – Providing unified framework to effectively address two types of HDR

Fig. 1. HDR regions exhibiting abrupt and significant radiance change (upper right) and smoother local transition (lower right).
– Window-based tone mapping method
  • Solving global optimization problem
  • Satisfying local constraints
– Use of linear function in each window
  • Naturally suppressing strong edges while retaining weak edges
  • Providing flexibility for image quality adjustments with only a few parameters
– Tone enhancement of ordinary images
  • Solving same optimization problem
Previous work

- Global tone mapping operators
  - Two main categories
    - Histogram based algorithms
    - Sigmoid transfer function based algorithms
  - Proposal of other non-linear global operators
    - Global tone reproduction curves with different measures
      - Computationally efficient way
    - Producing visually satisfactory results for a large examples
    - Failing for HDR images where strong local contrast is present
      - A local region contains both high and low radiance values
Local tone mapping operators

- Segmentation of HDR image
  - Applying different tone mapping curves to the segments
- Use of layers
  - Reflectance image and illuminance image
  - Base layer and detail layer
  - Decomposing HDR images into several layers
- Other methods
  - Edge-preserving multi-scale image decomposition method
  - Gradient domain dynamic range compression
Algorithm and implementation

◆ HDR compression operator
  – Computation of radiance map in LDR from radiance map of input HDR image

◆ Correctly retaining the local structures
  – Considering pixel’s neighborhood in local region
  – Local linear function

\[ I^l(j) = p_i I^h(j) + q_i, \quad j \in w_i \]  

(1)

where \( w_i \) is square window containing with pixel \( i \) in center,
\( p_i \) is slope of linear function, and
\( q_i \) is parameter determined base radiance level.
• Operation of coefficients $p_i$ and $q_i$

Fig. 2. Illustration of the local window operator. (a) and (b) show two configurations that $I^h$ undergoes linear mappings (represented as the slanted dashed lines) with different coefficients $p$ and $q$ to produce $I^l$. $q$ controls the base radiance level and $p$ manipulates the local contrast in the window. Setting large $p$ makes the local details in $I^l$ be enhanced, as shown in (a), whereas a small $p$ suppresses local high contrast, as shown in (b).
◆ Reconstructing image

- Minimizing Eq. (2)

\[
\sum_i \sum_{j \in w_i} \left( I^l(j) - p_i I^h(j) - q_i \right)^2
\]  

(2)

where \( i \) sums over all pixels in the HDR image.

- Zero error exist in minimizing Eq. (2) \( p_i = 1 \) and \( q_i = 0 \) for all \( i \) \( \Rightarrow \) \( I^l = I^h \)

- Need of additional constraints

- Proposal of \( p_i \) value guidance in each window

\[
f = \sum_i \left( \sum_{j \in w_i} \left( I^l(j) - p_i I^h(j) - q_i \right)^2 + \varepsilon c_i^{-2} (p_i - c_i)^2 \right)
\]  

(3)

where \( c_i \) is pre-set positive value to guide modification of local contrast, \( \varepsilon \) is a weight to balance the two terms.
Optimization and implementation

– Expression of minimizing Eq. (3)

\[
\arg\min_{p,q,I^l} f = \arg\min_{I^l} \sum_i \arg\min_{p_i,q_i} f_i,
\]

where \( f_i = \sum_{j \in W_i} \left( I_i^l(j) - p_i I_i^h(j) - q_i \right)^2 + \varepsilon c_i^{-2} (p_i - c_i)^2 \) \( (4) \)

• First computing \( (p_i^*, q_i^*) \)
  – Optimal solution of \( (p_i, q_i) \)
• Then computing optimal \( \hat{I}^l \)

– Reconstructing RGB channels

\[
I_k^l(i) = \left( I_k^l(i) / I_i^h(i) \right)^s \times \hat{I}^l(i), \quad k \in \{r, g, b\}
\]

where \( s \) is saturation factor. \( s \in [0.4, 0.6] \)
– Local window size
  • Suitable widow size $3 \times 3$
    – Edge sharpness and computational efficiency

*Fig. 3.* Tone mapping results with window sizes (a) $3 \times 3$, (b) $7 \times 7$, and (c) $15 \times 15$. (d)-(f) Close-ups of (a)-(c) respectively.
Guidance map configuration in HDR compression

◆ Definition of the guidance map $c$

- Determining quality of HDR compression

$$c_i = \left( \mu_i^{\beta_1} \sigma_i^{\beta_2} I^h(i)^{\beta_3} + \kappa \right)^{-1}$$

where $\mu_i$ is local mean in window,
$\sigma_i$ is standard deviation, and
$\kappa$ is small weight to prevent $c_i$ from being divided by zero.
$\beta_1 \in [0.4 \sim 0.9], \beta_2 \in [0.1 \sim 0.4], \text{ and } \beta_3 = 0.1$

Fig. 4. “Window” example. (a) The input HDR image (image courtesy of Shree Nayar). (b) The truncated variance map of $I^h$ on all local windows $w$. (c) The truncated map of mean values of $I^h$. (d) The truncated radiance map $I^h$. 
Fig. 5. Illustration of the guidance map configuration. (a) The tone mapping result when $c_i$ is set inversely proportional to the local standard deviation of $I^h$ in each window. There are visible artifacts on the wall. (b) Magnified guidance map $c$. The noise is amplified. (c) The tone mapping result when $c_i$ is configured according to (6). (d) Magnified guidance map. Note that the noise is suppressed.

\[ c_i = \left( \sigma_i^{\beta_2} + \kappa \right)^{-1}, \quad \beta_2 = 0.75 \]

\[ c_i = \left( \mu_i^{\beta_1} \sigma_i^{\beta_2} I^h(i)^{\beta_3} + \kappa \right)^{-1} \]
– Construction of guidance map $c$

$$c_i = \left( \mu_i^{\beta_1} \sigma_i^{\beta_2} I^h(i)^{\beta_3} + \kappa \right)^{-1}$$
– Preserving details in both bright and dark regions
  • Balance local contrast and radiance values

Fig. 7. Lamp desk. (a) An input HDR image. (b) The guidance map \( c \) generated from (a) using our method. (c) A tone-mapping result by our method. (d) The guidance map \( c \) generated from (c) using the same parameters. It is structurally flat comparing to the map shown in (b). (e)-(h) Close-ups of (a)-(d).
◆ Parameter settings

- Tuning parameters $\beta_1$ and $\beta_2$
- Sum of three $\beta$'s
  - Larger value makes the result visually flatter
- Increasing $\beta_2$
  - Enhancement of compression on strong edges
- Small modifications to $\beta_1$ and $\beta_2$
  - No largely affect thee result
Fig. 8. HDR compression with different parameter settings. (a) shows the original HDR image, displayed with linear scaling. (b)-(e) The tone mapping results with different parameter settings. Close-ups are shown in (f)-(i).

Fig. 9. HDR compression with different settings of $\beta_1$ and $\beta_2$. (a) The input HDR image. (b) Visualization of the map $\log_2(\mu+1)$. (c) Visualization of the map $\log_2(\sigma+1)$. (d)-(f) The tone mapping results with different parameter settings. We magnify two regions and compare them in (g)-(l). Local contrast is enhanced more significantly using larger $\beta_2$. 
Fig. 10. HDR compression with different $\beta_1$’s. (a) shows the HDR images under low exposures. (b) shows the HDR images under high exposures. (c)-(f) show our HDR compression results. In all these results, we fix $\beta_2 = 0.1$. Since $\beta_1 + \beta_2$ controls the overall compression ratio, larger $\beta_1 + \beta_2$ makes the result exhibit more details in both bright and dark regions.
◆ Running time
  – Using Matlab and Core2Duo 2.4Ghz CPU

**Fig. 11.** Plot of running time of our tone mapping algorithm with respect to different image resolutions.
More results and comparison

Fig. 12. Memorial Church. (a) Tone mapping result of Durand and Dorsey [7]. (b) Tone mapping result of Fattal et al. [9]. (c) Result by the multiscale method using one aggregated gain map [16]. (d) Our result with parameters $\beta_1 = 0.7$, $\beta_2 = 0.2$, $\beta_3 = 0.1$. (e)-(h) Magnified regions from (a)-(d) respectively. HDR image courtesy of Paul Debevec.
Fig. 13. Chairs. (a) Our tone mapping result with parameters $\beta_1 = 0.6$, $\beta_2 = 0.2$, and $\beta_3 = 0.1$. (b) The tone mapping result published in the project web page of [9]. HDR image courtesy of Shree Nayar.
Fig. 14. Street. HDR images © Industrial Light & Magic. The top row shows the input HDR image under different exposures. The bottom row shows our HDR compressed image result and its close-ups, with parameters $\beta_1 = 0.6$, $\beta_2 = 0.2$, and $\beta_3 = 0.1$. The radiance of the sun is largely reduced.
Fig. 15. More high dynamic range compression results by our method. The parameters that produce these results are identical: $\beta_1 = 0.5$, $\beta_2 = 0.2$, and $\beta_3 = 0.1$.

Fig. 16. Comparison with the result of Farbman et al. [8]. (a) The tone mapped image shown in [8]. (b) Our tone mapping result.
Applications

◆ Ordinary image enhancement
  – Performance of automatic enhancement of ordinary LDR images
    • Improvement of visibility of dark and saturated regions
  – Much lower contrast ordinary image
    • Setting smaller values of $\beta_1$, $\beta_2$, and $\beta_3$
      – $\beta_1 = 0.4$, $\beta_2 = 0.2$, and $\beta_3 = 0.05$
        » Work well for most examples
Fig. 17. Ordinary image enhancement. (a) and (c) The original images. (b) and (d) The automatically enhanced images showing more balanced local details and global luminance.
HDR image synthesis

- Method for synthesizing an HDR image from a single LDR image
- Minimizing in HDR compression model

$$ f = \sum_{j \in W_i} \left( I^l(j) - p_i I^h(j) - q_i \right)^2 + \varepsilon c_i^{-2} (p_i - c_i)^2 $$ (7)

- Expression in LDR expansion

$$ p_i^2 \left( \sum_{j \in W_i} \left( I^h(j) - \frac{1}{p_i} I^l(j) - \frac{q_i}{p_i} \right)^2 + \varepsilon c_i^{-2} \left( 1 - \frac{c_i}{p_i} \right)^2 \right) $$ (8)
– Computation simplicity

• Approximation of weight \( p_i^2 \) in (8) by \( c_i^2 \)

\[
E' = \sum_i c_i^2 \left( \sum_{j \in w_i} \left( I^h(j) - \frac{1}{p_i} I^l(j) + \frac{q_i}{p_i} \right)^2 + \varepsilon c_i^{-2} \left( 1 - \frac{c_i^2}{p_i} \right)^2 \right)
\]

\[
= \sum_i c_i^2 \left( \sum_{j \in w_i} \left( I^h(j) - \frac{1}{p_i} I^l(j) + \frac{q_i}{p_i} \right)^2 + \varepsilon (c_i^{-1} - p_i^{-1})^2 \right) \tag{9}
\]

• Denoting \( p_i' = 1/p_i, \ q_i' = -q_i/p_i, \) and \( c_i' = 1/c_i \)

\[
f' = \min_{p,q,I^h} \sum_i c_i'^{-2} \left( \sum_{j \in w_i} \left( I^h(j) - p_i' I^l(j) - q_i' \right)^2 + \varepsilon (p_i' - c_i')^2 \right) \tag{10}
\]

\[
c_i' = \left( \mu_i' \beta_i \sigma_i \beta_2 I^l(i) \beta_3 + \kappa \right), \quad \kappa = 1
\]

\[
\beta_1 \in [0.4 \sim 0.9], \beta_2 \in [0.1 \sim 0.4], \) and \( \beta_3 = 0.1
\]
Fig. 18. Image enhancement with different parameter settings. We fix $\beta_3 = 0.05$ in this example. Larger parameter values enhance more structural details.

Fig. 19. HDR image synthesis.
(a) The input LDR image $I'$ tone mapped from an HDR image $I^h$ displayed in (b).
(b) The original HDR image $I^h$ displayed under different exposures.
(c) The re-synthesized HDR image $I^{h'}$ by our algorithm (displayed under different exposures).
(d) The synthesized HDR image using the LDR2HDR method [22].
(e) The synthesized HDR image using linear scaling.
Fig. 20. Indoor example. (a) The input LDR image $I^l$ compressed from an HDR image. (b) The original HDR image $I^h$ shown under different exposures. (c) The re-synthesized HDR image $I^h$ from $I^l$ shown under different exposures.

Fig. 21. Night view example. (a) The input LDR image. (b)-(d) Our synthesized HDR scene, shown under different exposures.
Discussion and conclusion

Introducing novel HDR compression method

- Globally non-linear method
  - Use of overlapping window-based linear functions
    - Reconstruct image radiance
- Effectively suppressing global contrast
  - Preserving local image structure details
- Focus on automatic dynamic range compression
  - No difficult to add user interactions
Continuous convex function $f_i$

- Computation of $(p_i^*, q_i^*)$
  - Optimal solution of $(p_i, q_i)$

\[
\left. \frac{\partial f_i}{\partial p_i} \right|_{p_i=p_i^*, q_i=q_i^*} = 2\varepsilon c_i^{-2} (p_i^* - c_i) + \sum_{j \in w_i} 2 \left( I^l(j) - p_i^* I^h(j) - q_i^* \right) \cdot (-I^h(j)) = 0 \tag{11}
\]

\[
\left. \frac{\partial f_i}{\partial q_i} \right|_{p_i=p_i^*, q_i=q_i^*} = \sum_{j \in w_i} -2 \left( I^l(j) - p_i^* I^h(j) - q_i^* \right) = 0 \tag{12}
\]
- **Matrix form**

\[ H_i \cdot \left[ \begin{array}{c} p_i^* \\ q_i^* \end{array} \right]^T = \eta_i \]  

where

\[
H_i = \left[ \begin{array}{cc}
\varepsilon c_i^{-2} + \sum_{j \in w_i} I^h(j)^2 & \sum_{j \in w_i} I^h(j) \\
\sum_{j \in w_i} I^h(j) & \sum_{j \in w_i} 1
\end{array} \right]
\]

and

\[
\eta_i = \left[ \begin{array}{c}
\varepsilon c_i^{-2} + \sum_{j \in w_i} I^h(j) \cdot I^l(j) \\
\sum_{j \in w_i} I^l(j)
\end{array} \right].
\]

- **Solving (13)**

\[
\left[ \begin{array}{c} p_i^* \\ q_i^* \end{array} \right]^T = H_i^{-1} \cdot \eta_i = \frac{1}{m_i \cdot \Delta_i} \left[ \begin{array}{cc}
1 & -\mu_i \\
-\mu_i & \Delta_i + \mu_i^2
\end{array} \right] \cdot \eta_i
\]

where

\[
m_i = \sum_{j \in w_i} 1, \quad \mu_i = \frac{1}{m_i} \sum_{j \in w_i} I^h(j), \quad \text{and} \quad \Delta_i = \sigma_i^2 + \left( \frac{\varepsilon c_i^{-2}}{m_i} \right)
\]
Similarly computing \( \hat{I}^l \), optimal solution of (3)

\[
\frac{\partial f}{\partial I^l(k)}\bigg|_{I^l = \hat{I}^l} = \sum_{i|k \in w_i} 2\left(\hat{I}^l(k) - p_i^* I^h(k) - q_i^* \right) = 0
\]  

Combining (14) and (15)

\[
\frac{\partial f}{\partial I^l(k)}\bigg|_{I^l = \hat{I}^l} = \sum_{i|k \in w_i} \left(\hat{I}^l(k) - \frac{1}{m_i \Delta_i} (\epsilon c_i^{-1} I^h(k) 
+ I^h(k) \sum_{j \in w_i} I^h(j) \hat{I}^h(j) - \mu_i I^h(k) \sum_{j \in w_i} I^l(j) - \mu_i \epsilon c_i^{-1}
- \mu_i \sum_{j \in w_i} I^h(j) \hat{I}^h(j) + (\Delta_i + \mu_i^2) \sum_{j \in w_i} I^l(j))\right)
= 0
\]

Linear combination of \( \hat{I}^l \)'s
– Rewrite in a form of large linear system

\[ S \cdot \hat{I}^l = B \]  \hspace{1cm} (17)

where

\[ s_{kj} = \frac{\partial^2 f}{\partial I^l(k) \partial \hat{I}^l(j)} \]

\[ = \sum_{i \in \{k,j\} \subseteq w_i} \left( \delta_{kj} - \frac{1}{m_i \Delta_i} \left( (I^h(k) - \mu_i) \left( I^h(j) - \mu_i \right) + \Delta_i \right) \right) \]

\[ b_k = \sum_{i \in \{k,j\} \subseteq w_i} \frac{\epsilon}{m_i \Delta_i c_i} \left( I^h(k) - \mu_i \right) \]  \hspace{1cm} (18)
Fig. 22. Illustration of the region containing pixel $i$. All windows $w$ with size $3 \times 3$ and containing $i$ must be within the yellow region. There are at most 24 pixels in relation to $i$ by local windows $w$. Since there are $N$ pixels in an image, the total number of the non-zero elements in matrix $S$ will be less than $25 \times N$. 