Spectral Sensitivity Optimization of Color Image Sensors Considering Photon Shot Noise

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Flow chart

1. Definition of photon shot noise in image sensor
2. Modeling of color transformation matrix and photon shot noise
3. Definition of optimization metric for evaluation of error
4. Selecting color transformation matrix and color filter set through experiments
Abstract

◆ Optimization of digital camera color filters
  – Achievement of high color accuracy and low image noise

◆ Using RGB and considering photon shot noise
  – Longer (620 to 630nm) peak wavelength of R channel
    • Reducing noise fluctuation along a* axis and
    • Reducing color accuracy
  – Tradeoff between image noise and color accuracy
    • Leading to four-channel camera consisting of two R sensors and G and B
Development of digital still cameras

- Increase of resolution
  - Improvement of image quality
  - Constant image sensor’s size
  - Use of smaller pixel size
    - Smaller amount of photon energy
    - Increasing likelihood of noisy images
      » Inherent uncertainty in photon detection
        (photon shot noise)
  - Increasing importance of low noise sensor and image processing
Noise of image sensor

- Signal independent
  - Dark current noise and read-out random noise
  - Good performance of CMOS image sensor

- Signal dependent
  - Photon shot noise
    - Inherent noise in photon detection
    - Decreasing signal-to-noise ratio
      » Due to pixel pitch decrease and/or exposure index increase
Transforming signal into proper color reproduction

- Depending on sensor’s sensitivities
- Modulating noise components
- Design of spectral sensitivity
  - Considering both colorimetric and noise performance
– Methods of the past
  • Neugebauer or Vora
    – Not considering any noise
  • Vrhel, Sharma, and Trussell
    – Not considering photon shot noise
  • Quan, Mizukura, Katoh, and Nishio
    – Modeling photon shot noise for certain condition empirically

– This research
  • Evaluating RGB and CMY filter sets
    – Noise variance following color transformation
  • Optimization of spectral sensitivities
    – Leading to highest color accuracy, lowest image noise, or compromise of the two
Color transformation model

- Spectral sensitivity model of RGB sensors

  - Use of Gaussian function

\[
S_i(\lambda) = \exp \left[ -\frac{(\lambda - \lambda_i)^2}{w_i^2} \right]
\]

  where \( \lambda \) is wavelength, 
  \( \lambda_i \) is peak wavelength, 
  \( w \) is width, and 
  \( i = R, G, B \).

  - Combination of IR cut-off filter

Fig. 1. IR cut filter model
– Spectral sensitivity model of CMY sensors

• Use of Gaussian function

\[
C(\lambda) = \begin{cases} 
0, & \text{if } \lambda > \lambda_C \\
\exp \left[ -\frac{(\lambda - \lambda_C)^2}{w_C^2} \right], & \text{if } \lambda \leq \lambda_C 
\end{cases}
\]

\[M(\lambda) = 1 - \exp \left[ -\frac{(\lambda - \lambda_M)^2}{w_M^2} \right] \]

\[Y(\lambda) = \begin{cases} 
\exp \left[ -\frac{(\lambda - \lambda_Y)^2}{w_Y^2} \right], & \text{if } \lambda \geq \lambda_Y \\
0, & \text{if } \lambda < \lambda_Y 
\end{cases}
\]
– Color transformation from RGB or CMY to XYZ

\[
\begin{align*}
\left( \frac{X}{X_n} \right) &= \begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{pmatrix} \begin{pmatrix} C_1 / C_{1,n} \\ C_2 / C_{2,n} \\ C_3 / C_{3,n} \end{pmatrix} \\
\frac{Y}{Y_n} &= \frac{Z}{Z_n}
\end{align*}
\]

(5)

\[
m_1 + m_2 + m_3 = m_4 + m_5 + m_6 = m_7 + m_8 + m_9 = 1
\]

(6)

where \( m_1 \) to \( m_9 \) are coefficients of color transformation matrix, \( C_1, C_2, C_3, \) are either R, G, B or C, M, Y, and suffix \( n \) means value of white.
Noise model

- Variance of photon shot noise
  - Proportional to photon count
- Photon count $n_i$

\[
 n_i = \int \frac{I(\lambda)R(\lambda)tl^2}{hc/\lambda} \cdot Q_i(\lambda) d\lambda
\]  

where $I(\lambda)$ is spectral irradiance,
$R(\lambda)$ is spectral reflectance ratio of object,
$Q_i(\lambda)$ is spectral quantum efficiency of image sensor,
$t$ [s] is exposure time, $l^2$ is pixel area (pixel pitch is $l$ [m]),
h is Planck’s constant, $c$ is speed of light, and
$i = R,G,B$ or $i = C,M,Y$. 
From Appendix A

\[ n_i = \frac{l^2}{I_{EI}} \cdot \int \frac{55.6}{683} \int I_0(\lambda)V(\lambda)d\lambda \cdot \frac{I_0(\lambda)R(\lambda)}{hc / \lambda} \cdot Q_i(\lambda)d\lambda \]  

(8)

where \( I_{EI} \) is exposure index,
\( I_0 \) is relative spectral power distribution of illuminant, and
\( V(\lambda) \) is spectral luminous efficiency function.

Photon energy on a pixel

\[ P_E = \frac{(l \times 10^{-6})^2}{I_{EI} / 100} \cdot Q_{max} \]  

(9)

Table 1. Exposure indices corresponding to the parameter \( P_E \) and pixel pitch when \( Q_{max} = 1 \).
- Probability distribution of signal when evaluating noise
  - Treating as Gaussian random variable
- Standard deviation

$$\sigma_i = \sqrt{n_i + n_d^2}$$  \hspace{1cm} (10)

where $n_i$ and $n_d$ are photon count and noise count.

- Probability density of observed signal

$$P_i(x) = \frac{1}{\left[2(n_i + n_d^2)\right]^{1/2}} \exp \left[-\frac{(x - n_i)^2}{2(n_i + n_d^2)}\right]$$  \hspace{1cm} (11)
RGB filter set

- Two steps for evaluating RGB spectral sensitivities
  - Optimization of color transformation matrix
    - Minimizing average CIE94 of Macbeth ColorChecker under CIE illuminant D65 or A
  - Calculating average photon counts of each RGB channel
    - Use of Eq. (8)
    - Evaluating fluctuations of outputs
      » Application of Monte-Carlo simulation method
    - Sampling outputs according to probability distributions of photons at each RGB channel
      » Use of Eq. (11)
- Noise evaluation
  - Average fluctuation over Macbeth ColorChecker, $\bar{\sigma}_{94}$

- Spectral sensitivity optimization
  - Multidimensional nonlinear optimization
  - Applying brute force approach for ease of implementation
    - Parameters of spectral sensitivities, $\lambda_i$ and $w_i$
      - Totaling 16,500 sets each 10-nm step
        - $\lambda_R = 590$ to 690,
        - $\lambda_G = 540$ to 560,
        - $\lambda_B = 430$ to 460, and
        - $w_{R,G,B} = 30$ to 70
• Calculating $\Delta E_{94}^*$ and $\bar{\sigma}_{94}$ from all sets

• Definition of optimization metric

$$m = \sqrt{\Delta E_{94}^* + \alpha \bar{\sigma}_{94}^2}$$

(12)

where $\alpha$ is weighting parameter.

Table 2. Optimized sensitivity parameters according to $\alpha$ ($P_E = 0.27$) under CIE illuminant D65.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda_R$</th>
<th>$\lambda_G$</th>
<th>$\lambda_B$</th>
<th>$\omega_R$</th>
<th>$\omega_G$</th>
<th>$\omega_B$</th>
<th>$\Delta E_{94}^*$</th>
<th>$\bar{\sigma}_{94}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>600</td>
<td>550</td>
<td>450</td>
<td>60</td>
<td>50</td>
<td>30</td>
<td>0.24</td>
<td>8.14</td>
</tr>
<tr>
<td>0.125</td>
<td>620</td>
<td>540</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>0.69</td>
<td>6.38</td>
</tr>
<tr>
<td>0.25</td>
<td>630</td>
<td>540</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>1.03</td>
<td>6.16</td>
</tr>
<tr>
<td>0.5</td>
<td>640</td>
<td>540</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>1.38</td>
<td>6.02</td>
</tr>
<tr>
<td>1.0</td>
<td>650</td>
<td>550</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>40</td>
<td>2.03</td>
<td>5.79</td>
</tr>
</tbody>
</table>
Fig. 2. Optimized RGB QE curves and CMFs derived from them under CIE illuminant D65 ($\alpha=0$).

Fig. 3. Optimized RGB QE curves and CMFs derived from them under CIE illuminant D65 ($\alpha=0.25$).
Fig. 4. Color reproduction error and noise in the a*b* plane of RGB sensors under CIE illuminant D65 [(a) $\alpha=0$, (b) $\alpha=0.25$].

Table 3. Optimized sensitivity parameters according to $\alpha$ ($P_E =0.27$) under CIE illuminant A.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda_R$</th>
<th>$\lambda_G$</th>
<th>$\lambda_B$</th>
<th>$\omega_R$</th>
<th>$\omega_G$</th>
<th>$\omega_B$</th>
<th>$\Delta E_{94}$</th>
<th>$\bar{\sigma}_{94}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>600</td>
<td>550</td>
<td>450</td>
<td>60</td>
<td>50</td>
<td>30</td>
<td>0.27</td>
<td>8.97</td>
</tr>
<tr>
<td>0.125</td>
<td>620</td>
<td>540</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>40</td>
<td>0.68</td>
<td>7.49</td>
</tr>
<tr>
<td>0.25</td>
<td>630</td>
<td>540</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>40</td>
<td>1.02</td>
<td>7.27</td>
</tr>
<tr>
<td>0.5</td>
<td>640</td>
<td>550</td>
<td>450</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>1.63</td>
<td>6.96</td>
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<td>450</td>
<td>50</td>
<td>40</td>
<td>40</td>
<td>1.93</td>
<td>6.83</td>
</tr>
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</table>
Fig. 5. Optimized RGB QE curves and CMFs derived from them under CIE illuminant A ($\alpha=0$).

Fig. 6. Optimized RGB QE curves and CMFs derived from them under CIE illuminant A ($\alpha=0.25$).
Fig. 7. Color reproduction error and noise in the a*b* plane of RGB sensors under CIE illuminant A [(a) α=0, (b) α=0.25].

Table 4. Noise amplitude of RGB filter in L*a*b* space under CIE illuminants D65 and A.

<table>
<thead>
<tr>
<th>α</th>
<th>CIE illuminant D65</th>
<th>CIE illuminant A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{\sigma}_{L^*}$</td>
<td>$\bar{\sigma}_{a^*}$</td>
</tr>
<tr>
<td>0.0</td>
<td>1.86</td>
<td>12.12</td>
</tr>
<tr>
<td>0.25</td>
<td>2.21</td>
<td>7.30</td>
</tr>
</tbody>
</table>
◆ CMY filter set
  – Evaluating in the same way as RGB spectral sensitivities
    • Parameters of spectral sensitivities, $\lambda_{C,M,Y}$ and $w_{C,M,Y}$
      – Totaling 13,500 sets each 10-nm step
        $\lambda_C = 560$ to 640, $\lambda_M = 530$ to 550
        $\lambda_Y = 440$ to 470, $w_{C,M,Y} = 40$ to 80

Table 5. Optimized CMY parameters according to $\alpha$ ($P_E = 0.27$) under CIE illuminant D65.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda_C$</th>
<th>$\lambda_M$</th>
<th>$\lambda_Y$</th>
<th>$\omega_C$</th>
<th>$\omega_M$</th>
<th>$\omega_Y$</th>
<th>$\Delta E_{94}$</th>
<th>$\tilde{\gamma}_{94}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>570</td>
<td>540</td>
<td>470</td>
<td>50</td>
<td>40</td>
<td>60</td>
<td>1.60</td>
<td>14.23</td>
</tr>
<tr>
<td>0.125</td>
<td>610</td>
<td>540</td>
<td>470</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td>1.98</td>
<td>8.76</td>
</tr>
<tr>
<td>0.25</td>
<td>610</td>
<td>540</td>
<td>470</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td>1.98</td>
<td>8.76</td>
</tr>
<tr>
<td>0.5</td>
<td>610</td>
<td>540</td>
<td>470</td>
<td>40</td>
<td>70</td>
<td>40</td>
<td>2.17</td>
<td>8.66</td>
</tr>
<tr>
<td>1.0</td>
<td>620</td>
<td>540</td>
<td>470</td>
<td>40</td>
<td>70</td>
<td>40</td>
<td>2.25</td>
<td>8.62</td>
</tr>
</tbody>
</table>
Fig. 8. Optimized CMY QE curves and CMFs derived from them under D65 ($\alpha=0$).

Fig. 9. Optimized CMY QE curves and CMFs derived from them under D65 ($\alpha=0.25$).
Table 6. Optimized CMY parameters according to $\alpha$ ($P_E = 0.27$) under CIE illuminant A.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda_C$</th>
<th>$\lambda_M$</th>
<th>$\lambda_Y$</th>
<th>$\omega_C$</th>
<th>$\omega_M$</th>
<th>$\omega_Y$</th>
<th>$\Delta E_{94}$</th>
<th>$\tilde{\sigma}_{94}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>610</td>
<td>530</td>
<td>470</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>1.62</td>
<td>16.56</td>
</tr>
<tr>
<td>0.125</td>
<td>560</td>
<td>540</td>
<td>450</td>
<td>60</td>
<td>70</td>
<td>40</td>
<td>2.26</td>
<td>11.22</td>
</tr>
<tr>
<td>0.25</td>
<td>560</td>
<td>550</td>
<td>450</td>
<td>40</td>
<td>70</td>
<td>40</td>
<td>2.66</td>
<td>10.78</td>
</tr>
<tr>
<td>0.5</td>
<td>560</td>
<td>550</td>
<td>460</td>
<td>40</td>
<td>80</td>
<td>40</td>
<td>2.84</td>
<td>10.62</td>
</tr>
<tr>
<td>1.0</td>
<td>570</td>
<td>550</td>
<td>460</td>
<td>40</td>
<td>80</td>
<td>40</td>
<td>2.87</td>
<td>10.61</td>
</tr>
</tbody>
</table>

Fig. 10. Color reproduction error and noise in the $a^*b^*$ plane of CMY sensors under D65 [(a) $\alpha=0$, (b) $\alpha=0.25$].
Fig. 11. Optimized CMY QE curves and CMFs derived from them under A ($\alpha=0$).

Fig. 12. Optimized CMY QE curves and CMFs derived from them under A ($\alpha=0.25$).
Fig. 13. Color reproduction error and noise in the $a^*b^*$ plane of CMY sensors under A [(a) $\alpha=0$, (b) $\alpha=0.25$].
Fig. 14. Color shift due to noise in $a^*b^*$ plane of CMY sensors under A [(a) $\alpha=0$, (b) $\alpha=0.25$].

Table 7. Noise amplitudes of CMY filter in $L^*a^*b^*$ space under CIE illuminant D65 and A.

<table>
<thead>
<tr>
<th></th>
<th>CIE illuminant D65</th>
<th>CIE illuminant A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\bar{\sigma}_L$</td>
<td>$\bar{\sigma}_{a^*}$</td>
</tr>
<tr>
<td>0.0</td>
<td>5.20</td>
<td>22.84</td>
</tr>
<tr>
<td>0.25</td>
<td>2.58</td>
<td>11.48</td>
</tr>
</tbody>
</table>
Effect of color transformation matrix

- Imaging gray object under daylight

  - Signal variance \((\Delta L^*)^2, (\Delta a^*)^2, (\Delta b^*)^2\)

\[
(\Delta L^*)^2 \approx \left(\frac{116A}{3}\right)^2 (m_{21} + m_{22} + m_{23})^2
\]  
(13)

\[
(\Delta a^*)^2 \approx \left(\frac{500A}{3}\right)^2 \left\{ (m_{11} + m_{12} + m_{13}) - (m_{21} + m_{22} + m_{23}) \right\}^2
\]  
(14)

\[
(\Delta b^*)^2 \approx \left(\frac{200A}{3}\right)^2 \left\{ (m_{21} + m_{22} + m_{23}) - (m_{31} + m_{32} + m_{33}) \right\}^2
\]  
(15)
– Color transformation matrix of type $C_{\text{RGB}}$

$$M_{C, \text{RGB}} = \begin{pmatrix} 1.0044 & -0.2011 & 0.1966 \\ 0.3183 & 0.6652 & 0.0165 \\ -0.0166 & 0.0538 & 0.9628 \end{pmatrix}$$

(16)

- Relative amplitude of noise

$$|\Delta L^*| : |\Delta a^*| : |\Delta b^*| = \sqrt{813.34} : \sqrt{34833.33} : \sqrt{6137.78}$$

$$= 29 : 187 : 78$$

(17)
– Color transformation matrix of type $N_{RGB}$

$$
M_{N,RGB} = \begin{pmatrix}
0.5570 & 0.2907 & 0.1524 \\
0.2382 & 0.7616 & 0.0002 \\
-0.0047 & 0.0421 & 0.9626 \\
\end{pmatrix}
$$

(18)

• Relative amplitude of noise

$$
\Delta L^* : \Delta a^* : \Delta b^* = \sqrt{952.05} : \sqrt{9638.89} : \sqrt{6680.00}
$$

(19)

$$
= 31 : 98 : 82
$$

– Much smaller than $C_{RGB}$ in $a^*$
– Color transformation matrix of type $C_{CMY}$

$$
M_{C,CMY} = \begin{pmatrix}
-0.2131 & 0.5670 & 0.6461 \\
1.0006 & -1.3126 & 1.3120 \\
0.7376 & 0.6261 & -0.3637
\end{pmatrix}
$$

(20)

• Relative amplitude of noise

$$
|\Delta L^*| : |\Delta a^*| : |\Delta b^*| = \sqrt{6646.46} : \sqrt{151371.8} : \sqrt{29491.98} \\
= 82 : 389 : 172
$$

(21)

– Much larger than RGB filter
– Color transformation matrix of type $N_{CMY}$

$$M_{N,CMY} = \begin{pmatrix}
-0.2780 & 0.5037 & 0.7743 \\
0.5723 & -0.5057 & 0.9333 \\
0.8496 & 0.7483 & -0.5979
\end{pmatrix}$$

(22)

• Relative amplitude of noise

$$|\Delta L^*| : |\Delta a^*| : |\Delta b^*| = \sqrt{2174.35} : \sqrt{49088.31} : \sqrt{17751.04}$$

$$= 47 : 222 : 133$$

(23)

– Smaller than $C_{CMY}$
  » Much larger than RGB filter

Table 8. Average photon counts of white under CIE illuminant D65.

<table>
<thead>
<tr>
<th>Type</th>
<th>RGB filter</th>
<th>CMY filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>G</td>
</tr>
<tr>
<td>Type C</td>
<td>370.34</td>
<td>411.66</td>
</tr>
<tr>
<td>Type N</td>
<td>231.45</td>
<td>342.83</td>
</tr>
</tbody>
</table>
Four-channel color image sensor
  – Type $N_{RGB}$ filter of less noise fluctuation
    • Existing considerable error of estimated CMF
      – Observer metamerism
  – Measurement of MetaCow
    • One half of each cow
      – Assignment of spectral reflectance properties measured from GretagMacbeth ColorChecker
    • Other half
      – Assignment of highly metameric match to that reflectance for CIE illuminant D65 and 1931 CIE Standard Observer 2°
Fig. 15. MetaCow with type $C_{RGB}$ filter set: $\lambda_i=(600, 550, 450)$, $w_i=(60, 50, 30)$, and $P_E=0.27$. (Color online only.)

Fig. 16. MetaCow with type $N_{RGB}$ filter set: $\lambda_i=(630, 540, 450)$, $w_i=(50, 40, 30)$, and $P_E=0.27$. (Color online only.)
- Ideal filter set
  - Good noise performance without observer metamerism
  - Impossible with RGB filter set according to the result
- Propose of four-channel color filter set
  - G, B, and two R channels
    \[ \lambda_{R1} = 600, \lambda_{R2} = 630, \text{ and } w_{R1} = w_{R2} = 50 \]
    \[ \lambda_{G} = 550, w_{G} = 40, \lambda_{B} = 450, w_{B} = 30 \]
  - Property of type \( C_{RGB} \)
    \[ (\lambda_{R1}, \lambda_{G}, \lambda_{B}) = (600, 550, 450) \]
  - Property of type \( N_{RGB} \)
    \[ (\lambda_{R1}, \lambda_{G}, \lambda_{B}) = (630, 550, 450) \]
**Fig. 17.** MetaCow with type CN filter set: $\lambda_i=(630, 600, 540, 450)$, $w_i=(50, 50, 40, 30)$, and $P_E=0.27$. (Color online only.)

**Fig. 18.** MetaCow (part, $100 \times 100$ pixels) with type $C_{RGB}$, $N_{RGB}$, and CN, $P_E=0.27.$
Conclusion

◆ Optimizing spectral sensitivities of image sensor
  – Considering color reproduction and photon shot noise
    • RGB-type sensors better than CMY-types
  – Case of RGB filter sets
    • Suppressing noise along $a^*$ axis
      – R peak wavelength $\lambda_R$ longer than 600nm
        » Making approximation of CMFs worse
        » Reducing color accuracy
    • Overcoming by using two R channels
      – 600nm and longer than 600nm
        » Achieving high color accuracy and low image noise
<table>
<thead>
<tr>
<th>α</th>
<th>λ_R</th>
<th>λ_G</th>
<th>λ_B</th>
<th>ω_R</th>
<th>ω_G</th>
<th>ω_B</th>
<th>ΔE_{94}</th>
<th>σ_{94}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>600</td>
<td>550</td>
<td>450</td>
<td>60</td>
<td>50</td>
<td>30</td>
<td>0.24</td>
<td>8.14</td>
</tr>
<tr>
<td>0.125</td>
<td>620</td>
<td>540</td>
<td>450</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>0.69</td>
<td>6.38</td>
</tr>
<tr>
<td>0.25</td>
<td>630</td>
<td>540</td>
<td>450</td>
<td>50</td>
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<td>30</td>
<td>1.03</td>
<td>6.16</td>
</tr>
<tr>
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<td>5.79</td>
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\[ m = \left( \Delta E_{94}^* + \alpha \sigma_{94}^2 \right)^{1/2} \]

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