Three-dimensional Interconnection Scheme for Color Error Diffusion

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Abstract

◆ Purpose
  – Use of 3D error diffusion neural networks algorithm (EDNs) for color error diffusion
    • Related problems
      – Computation of binary halftones for each primary color—red, green, and blue
      – Minimization of frequency-weighted error in luminosity
    • EDN with 3D interconnection scheme
      – Solution of all problems in parallel
Introduction

- Classical error diffusion
  - 1D error diffusion
    - Scalar error diffusion
    - Leading undesirable visual artifacts
  - 2D error diffusion
    - Error diffusion neural network (EDN)
    - Symmetrical diffusion of error to all directions
  - Advantage of neural algorithm
    - Computation of all pixel quantization in parallel
    - Use of undirectional and symmetric filter
    - Reduction of artifacts
Error diffusion for color halftoning

– Previous methods
  • Kolpatzik-Bouman algorithm
    – Use of separable error filters in a luminance-chrominance space
  • Vector error diffusion algorithms
    – Performance of quantization step jointly for entire color vector
  • Shaked algorithm
    – Incorporation of MBVC (minimal brightness variation criterion) design rule
  • Making better color halftone textures than scalar versions
– Proposed method
  • New 3D interconnection scheme
    – 2D filters
      » Interconnection neighbor pixels within each color plane
    – Introduction of luminosity error diffusion filters
  • Luminosity
    – Quality of correspondance relative visual sensitivity of human eye to primaries
    – Correlation among the three color planes
  • Performance of tasks
    – Minimization of frequency-weighted error for three primaries
    – Minimization of luminosity error
Algorithms

◆ 2D grayscale EDN
  – Halftone algorithm

\[ y = h(x) \begin{cases} 1, & x = 0, 1 \\ 0, & \text{otherwise} \end{cases} \]

where \( x \) is a discretely sampled real-valued input.

– Energy function \( E(x, y) \)
  • Sum of square of spatial frequency-weighted error between \( x \) and \( y \)
  • Low value of \( E(x, y) \)
    » Good halftone
    » Minimization of \( E(x, y) \)
– Artificial neural network
  • Groups of interconnected processing neurons
    – Working in parallel
  • Input
    – Receiving from interconnected other neurons
  • Output
    – Passing to other neurons through interconnection weights
  • Weight
    – Interconnection strengths between neural pairs
  • Resemblance of pixels and neurons
    – Pixels of error diffusion
– The Hopfield-type neural network
  • Dynamic behavior of a $N$-neuron Hopfield-type neural network

\[
c \frac{du_i(t)}{dt} = -u_i(t) + \sum_{j} w_{i,j} \mathcal{F}[u_j(t)] + x_i
\]  

where  
\[ i = 1, 2, \ldots, N, \]
\( \mathcal{F}[\cdot] \) is monotonically increasing sigmoid function,  
\( x_i \) is input $N$-vector,  
\( c \) is scaling factor, and  
\( w_{i,j} \) is matrix of interconnection weights zero diagonal elements.

• Implication of Eq.(1)

\[
u_i = x_i + \sum_{j} w_{i,j} \mathcal{F}[u_j]
\]  

(2)
• Stable states of $N$ functions $y_i(t) = \mathcal{F}[\cdot]$
  – Local minima of energy function

$$E = -\frac{1}{2}y^T Wy - y^T x$$  \hspace{1cm} (3)$$

where $y$ is an $N$-vector of quantized states
$W$ is an $N \times N$ circularly symmetric matrix from $w_{i,j}$
– Error diffusion neural network

\[ u = W(y - u) + x \] (4)

• Equivalence of Hopfield network

\[ u = A(Wy + x) = AWy + Ax \] (5)

where \( A = (I + W)^{-1} \)

Fig. 1. Block diagram of a 2D error diffusion architecture.
• Energy function

\[
E(x,y) = -y^T(AW)y - 2y^T Ax
\]  
(6)

- Let \( k = y^Ty + x^T Ax \)

\[
E'(x,y) = E(x,y) + k
\]

\[
= -y^T(AW)y - 2y^T Ax + y^Ty + x^T Ax
\]

\[
= -y^T(I - A)y - 2y^T Ax + y^T y + x^T Ax
\]

\[
= y^T Ay - 2y^T Ax + x^T Ax
\]

\[
= (y - x)^T A(y - x)
\]

\[
= e^TAe
\]  
(7)
• Fourier transform of Eq.(7)

\[ \tilde{E}(x,y) = \sum_{k=0}^{n-1} \tilde{A}_k (\tilde{y}_k - \tilde{x}_k)^2 = \sum_{k=0}^{n-1} \tilde{A}_k \tilde{e}_k^2 \] (8)

where \( \tilde{A} \) is frequency weighting from \( A \).

• Modified Eq.(4)

\[ u_{n+1} = \alpha u_n + (1 - \alpha)[W(y - u_n) + x] \]

– \( \alpha \) less than 1 (here, \( \alpha = 0.99 \))
– Repeating until convergence
– 2D error diffusion interconnection weights
  • 2D finite impulse response (FIR) low-pass filter
    – Kernel size of 7x7
    – Design from windowing techniques
  • Frequency response of weight filter
    – Circularly symmetric
    – Minimization of directional artifacts
• Impulse response of ideal low-pass circularly symmetric filter

\[ h(n_1, n_2) = \frac{R \cdot J_1\left(\sqrt{n_1^2 + n_2^2}\right)}{2\pi \sqrt{n_1^2 + n_2^2}} \]

where \( 0 \leq R = 0.43\pi \leq \pi \) is cutoff frequency, \( J_1[\cdot] \) is Bessel function of first kind, and \( n_1, n_2 \) are two spatial dimensions.

• Use of Kaiser window with parameter \( \alpha \)

\[ w_k(n_1, n_2) = \begin{cases} I_0\left(\alpha \sqrt{1 - [(n_1^2 + n_2^2) / 25]}\right), & n_1^2 + n_2^2 \leq 25 \\ I_0^2[\alpha], & \text{otherwise} \end{cases} \]

Where \( I_0 \) is modified Bessel function.
• Result of impulse response and frequency response

\[
W_{RR} = W_{GG} = W_{BB} = \begin{bmatrix}
0.0005 & 0.0020 & 0.0052 & 0.0069 & 0.0052 & 0.0020 & 0.0005 \\
0.0020 & 0.0104 & 0.0249 & 0.0329 & 0.0249 & 0.0104 & 0.0020 \\
0.0052 & 0.0249 & 0.0584 & 0.0767 & 0.0584 & 0.0249 & 0.0052 \\
0.0069 & 0.0329 & 0.0767 & 0.0000 & 0.0767 & 0.0329 & 0.0069 \\
0.0052 & 0.0249 & 0.0584 & 0.0767 & 0.0584 & 0.0249 & 0.0052 \\
0.0020 & 0.0104 & 0.0249 & 0.0329 & 0.0249 & 0.0104 & 0.0020 \\
0.0005 & 0.0020 & 0.0052 & 0.0069 & 0.0052 & 0.0020 & 0.0005
\end{bmatrix}
\]

Fig. 2. (a) Impulse response; and (b) frequency response of the 7x7 error diffusion filter.
3D EDNs for color halftoning

- Three neurons per one pixel in EDN
- Description of Input pixel and output pixel

\[
\begin{align*}
<x_R, x_G, x_B> & \in [0,1]^3 \\
y_R, y_G, y_B & \in \{0,1\}^3
\end{align*}
\]

Fig. 3. 3D error diffusion neural network.
– Luminosity
  • Sensitivity of human eye to primaries
  • Luminosity of arbitrary pixel \( < r, g, b > \)

\[
l = \beta_R r + \beta_G g + \beta_B b
\]

Where \( \beta_R, \beta_G, \beta_B > 0 \) and \( \beta_R + \beta_G + \beta_B = 1. \)

– This paper

\[
\begin{align*}
\beta_R &= 0.30 \\
\beta_G &= 0.59 \\
\beta_B &= 0.11
\end{align*}
\]
• Color halftones of flat gray

\[ \sigma^2 = \frac{(0 - 0.25)^2 + (0 - 0.25)^2 + (1 - 0.25)^2 + (0 - 0.25)^2}{4} \approx 0.19 \]

(a) Identical gray pixels <0.25, 0.25, 0.25>

(b) Some possible color halftones

(c) Corresponding luminosity and variance:

\( \sigma^2 \approx 0.19 \quad 0.14 \quad 0.083 \quad 0.067 \quad 0.050 \)

Fig. 4. Luminosity variance of different color halftones of flat gray.
– Minimization of luminosity error
  • Way of minimization of luminosity error

\[
e = (\beta_R y_R + \beta_G y_G + \beta_B y_B) - (\beta_R x_R + \beta_G x_G + \beta_B x_B)
= \beta_R (y_R - x_R) + \beta_G (y_G - x_G) + \beta_B (y_B - x_B)
= \beta_R e_R + \beta_G e_G + \beta_B e_B
\]

– Let \( a_i = |\tilde{A}_i| \)

\[
E_l = \sum_i a_i |\tilde{e}|^2
= \sum_i a_i [\beta_R^2 (\tilde{e}_R)_i (\tilde{e}_R)_i^* + \beta_G^2 (\tilde{e}_G)_i (\tilde{e}_G)_i^* + \beta_B^2 (\tilde{e}_B)_i (\tilde{e}_B)_i^* \\
+ 2 \beta_R \beta_G (\tilde{e}_R)_i (\tilde{e}_G)_i^* + 2 \beta_G \beta_B (\tilde{e}_G)_i (\tilde{e}_B)_i^* + 2 \beta_B \beta_R (\tilde{e}_B)_i (\tilde{e}_R)_i^* ]
\]
• Transformation of matrix form

\[
x_c = \begin{bmatrix} x_R \\ x_G \\ x_B \end{bmatrix}, \quad y_c = \begin{bmatrix} y_R \\ y_G \\ y_B \end{bmatrix}
\]

– Transforming to monochrome

\[
x = L x_c \quad y = L y_c
\]  \hspace{1cm} (10)

where

\[
L = \begin{bmatrix}
\beta_R & 0 & \cdots & 0 & \beta_G & 0 & \cdots & 0 & \beta_B & 0 & \cdots & 0 \\
0 & \beta_R & \cdots & 0 & 0 & \beta_G & \cdots & 0 & 0 & \beta_B & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \beta_R & 0 & 0 & \cdots & \beta_G & 0 & 0 & \cdots & \beta_B
\end{bmatrix} = [\beta_R \mathbf{I} | \beta_G \mathbf{I} | \beta_B \mathbf{I}]
\]

• Generalization of Eq. (7)

\[
E_I = e_c^T (L^T AL) e_c
\]  \hspace{1cm} (11)

where \( e_c = y_c - x_c \)
– Minimizing error for each primary
• Energy function for an EDN in red primary

\[ E_{pr} = e_c^T (R^T A R) e_c \]

where \( R = [I|0|0] \)

• Solving by a single EDN
  – Three independent problems

\[ E_p = E_{pr} + E_{pg} + E_{pb} \]

\[ = [e_c^T (R^T A R) e_c] + [e_c^T (G^T A G) e_c] + [e_c^T (B^T A B) e_c] \]

\[ = [e_c^T (R^T A R + G^T A G + B^T A B) e_c] \]

where \( G = [0|I|0], \ B = [0|0|I] \)

(12)
– EDN 3D interconnection weights

• Similarity of Eqs. (11) and (12)

• Combination of luminosity and primary color energy terms

\[
E_c = E_p + E_l \\
= e_c^T (R^T AR + G^T AG + B^T AB) e_c \\
+ e_c^T (L^T AL) e_c \\
= e_c^T \tau [ \rho_R (R^T AR) + \rho_G (G^T AG) + \rho_B (B^T AB) \\
+ (L^T AL)] e_c
\]

where \( \rho_R, \rho_G, \rho_B > 0, \ \tau > 0 \)
• Energy function matrix

\[
\mathbf{A}_c = \tau \begin{bmatrix}
(\beta_R^2 + \rho_R) & \beta_R \beta_G & \beta_R \beta_B \\
\beta_R \beta_G & (\beta_G^2 + \rho_G) & \beta_G \beta_B \\
\beta_R \beta_B & \beta_G \beta_B & (\beta_B^2 + \rho_B)
\end{bmatrix} \ast \mathbf{A} \tag{13}
\]

• Solvation for the interconnection weights of EDN

\[
\mathbf{W}_c = \mathbf{A}_c^{-1} - \mathbf{I} = \mathbf{K}^{-1} \ast \mathbf{A}^{-1} - \mathbf{I} \tag{14}
\]

– Convergence of EDN

\[
\text{diag}(\mathbf{W}_c) = 0
\]

\[
\text{diag}(\mathbf{A}_c^{-1}) = \mathbf{I}
\]

\[
\text{diag}(\mathbf{K}^{-1} \ast \mathbf{A}^{-1}) = \mathbf{I} \tag{15}
\]
• Simplification of problem
  – Let

\[
\begin{align*}
  r &= 1 + \frac{\rho_R}{\beta_R^2}, \quad g = 1 + \frac{\rho_G}{\beta_G^2}, \quad \text{and} \quad b = 1 + \frac{\rho_B}{\beta_B^2} \\
  \rho_R &= (r - 1) \beta_R^2, \quad \rho_G = (g - 1) \beta_G^2, \quad \rho_B = (b - 1) \beta_B^2,
\end{align*}
\]

(16)

and

\[
K = \tau \begin{bmatrix}
  r \beta_R^2 & \beta_R \beta_G & \beta_R \beta_B \\
  \beta_R \beta_G & g \beta_G^2 & \beta_G \beta_B \\
  \beta_R \beta_B & \beta_G \beta_B & b \beta_B^2
\end{bmatrix}
\]
– Inverse $\mathbf{K}$

$$
\mathbf{K}^{-1} = \frac{1}{\tau} \frac{1}{2 - r - g - b + rgb}
\begin{bmatrix}
gb - 1 & 1 - b & 1 - g \\
\frac{\beta^2}{\beta_R} & \frac{\beta}{\beta_R \beta_G} & \frac{\beta}{\beta_R \beta_B} \\
1 - b & \frac{rb - 1}{\beta_R \beta_G} & \frac{1 - r}{\beta_R \beta_B} \\
\frac{1 - g}{\beta_R \beta_B} & \frac{1 - r}{\beta_G \beta_B} & \frac{rg - 1}{\beta_B^2}
\end{bmatrix}
$$

(18)

– From Eq. (14) \(\frac{gb - 1}{\beta_R^2} = \frac{rb - 1}{\beta_G^2} = \frac{rg - 1}{\beta_B^2} = d\)

(19)

where $d$ is an arbitrary positive value.
– Choose of constant $\tau$

\[
\tau = \frac{d}{2 - r - g - b + rgb}
\]  \hspace{1cm} (20)

– Transform of Eq. (18)

\[
K^{-1} = \begin{bmatrix}
1 & \frac{1-b}{d\beta_R\beta_G} & \frac{1-g}{d\beta_R\beta_B} \\
\frac{1-b}{d\beta_R\beta_G} & 1 & \frac{1-r}{d\beta_G\beta_B} \\
\frac{1-g}{d\beta_R\beta_B} & \frac{1-r}{d\beta_G\beta_B} & 1
\end{bmatrix}
\]  \hspace{1cm} (21)
• Computation of $r$
  - Independence of $\rho_R$ from Eq. (16)

• Computation of $g, b$

$$g = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$  \hspace{1cm} (22)

where

$$A = \beta_G^2 r, \quad B = \beta_B^2 - \beta_R^2 r^2 - \beta_G^2, \quad \text{and} \quad C = r(\beta_R^2 - \beta_B^2)$$

and

$$b = \frac{\beta_G^2 - \beta_R^2}{\beta_G^2 g - \beta_R^2 r}$$  \hspace{1cm} (23)
• Let \( k_{ij} \)
  – \( ij \)'th element of \( K^{-1} \)
  – Connection between red and green plane
    \[
    k_{12}(W + I)
    \]
  – Corresponding convolution kernel
    » Replacing center zero of monochrome kernel with a \( l \) and multiplying the whole by \( k_{12} = k_{21} \)
  – Influence of Error
    » Phase of patterns in others
  – Choose \( \rho_R = 2.0 \)
    \[
    \rho_G = 1.7, \quad \rho_B = 2.0, \quad k_{12} = -0.0842, \quad k_{13} = -0.0133, \quad \text{and} \quad k_{23} = -0.0299
    \]
• Result of cross plane interconnection weights

\[
\begin{bmatrix}
-0.0000 & -0.0002 & -0.0004 & -0.0006 & -0.0004 & -0.0002 & -0.0000 \\
-0.0002 & -0.0009 & -0.0021 & -0.0028 & -0.0021 & -0.0009 & -0.0002 \\
-0.0004 & -0.0021 & -0.0049 & -0.0065 & -0.0049 & -0.0021 & -0.0004 \\
-0.0006 & -0.0028 & -0.0065 & 0.0842 & -0.0065 & -0.0028 & -0.0006 \\
-0.0004 & -0.0021 & -0.0049 & -0.0065 & -0.0049 & -0.0021 & -0.0004 \\
-0.0002 & -0.0009 & -0.0021 & -0.0028 & -0.0021 & -0.0009 & -0.0002 \\
-0.0000 & -0.0002 & -0.0004 & -0.0006 & -0.0004 & -0.0002 & -0.0000
\end{bmatrix}
\]

\[W_{RG} = W_{GR} = \]

\[
\begin{bmatrix}
-0.0000 & -0.0001 & -0.0002 & -0.0002 & -0.0002 & -0.0001 & -0.0000 \\
-0.0001 & -0.0003 & -0.0007 & -0.0010 & -0.0007 & -0.0003 & -0.0001 \\
-0.0002 & -0.0007 & -0.0017 & -0.0023 & -0.0017 & -0.0007 & -0.0002 \\
-0.0002 & -0.0010 & -0.0023 & 0.0299 & -0.0023 & -0.0010 & -0.0002 \\
-0.0002 & -0.0007 & -0.0017 & -0.0023 & -0.0017 & -0.0007 & -0.0002 \\
-0.0001 & -0.0003 & -0.0007 & -0.0010 & -0.0007 & -0.0003 & -0.0001 \\
-0.0000 & -0.0001 & -0.0002 & -0.0002 & -0.0002 & -0.0001 & -0.0000
\end{bmatrix}
\]

\[W_{GB} = W_{BG} = \]

\[
\begin{bmatrix}
-0.0000 & -0.0000 & -0.0001 & -0.0001 & -0.0001 & -0.0000 & -0.0000 \\
-0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0003 & -0.0001 & -0.0000 \\
-0.0001 & -0.0003 & -0.0008 & -0.0010 & -0.0008 & -0.0003 & -0.0001 \\
-0.0001 & -0.0004 & -0.0010 & 0.0133 & -0.0010 & -0.0004 & -0.0001 \\
-0.0001 & -0.0003 & -0.0008 & -0.0010 & -0.0008 & -0.0003 & -0.0001 \\
-0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0003 & -0.0001 & -0.0000 \\
-0.0000 & -0.0000 & -0.0001 & -0.0001 & -0.0001 & -0.0000 & -0.0000
\end{bmatrix}
\]

\[W_{BR} = W_{RB} = \]

\[
\begin{bmatrix}
-0.0000 & -0.0000 & -0.0001 & -0.0001 & -0.0001 & -0.0000 & -0.0000 \\
-0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0003 & -0.0001 & -0.0000 \\
-0.0001 & -0.0003 & -0.0008 & -0.0010 & -0.0008 & -0.0003 & -0.0001 \\
-0.0001 & -0.0004 & -0.0010 & 0.0133 & -0.0010 & -0.0004 & -0.0001 \\
-0.0001 & -0.0003 & -0.0008 & -0.0010 & -0.0008 & -0.0003 & -0.0001 \\
-0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0003 & -0.0001 & -0.0000 \\
-0.0000 & -0.0000 & -0.0001 & -0.0001 & -0.0001 & -0.0000 & -0.0000
\end{bmatrix}
\]
Test Images

- Gray image and color image

Test images are (a) gray scale image that value is 77 and (b) color image.
Fig. 5.
(a) Grayscale image; (b) 2D grayscale EDN; (c) 3D interconnection weights When $\rho_R = 2.0$; (d) Floyd-Steinberg’s filter; (e) Kolpatzik-Bouman method; (f) Damera-Venkata vector error diffusion; and (g) Shaked’s MBVC.
Fig. 6. A radially averaged power spectrum in luminosity for halftone images of Figs. 5(b)-5(g).

- Average of power spectrum

<table>
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<td>from Fig. 5(b) – 2D filter</td>
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<td>from Fig. 5(c) – 3D filter</td>
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<td>from Fig. 5(d) – Floyd-Steinberg</td>
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<td>from Fig. 5(f) – Damera-Venkata</td>
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<td>from Fig. 5(g) – Shaked MBVC</td>
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Fig. 7.
(a) The original image; (b) 2D gray-scale EDN; (c) 3D interconnection weights when \( \rho_R = 2.0 \); (d) Floyd-Steinberg’s filter; (e) Kolpatzik-Bouman method; (f) Damera-Venkata vector error diffusion; and (g) Shaked’s MBVC.

- Halftone results of color image
• Radially averaged power spectrum

Fig. 8. Halftone image generated from 3D interconnection weights when (a) $\rho_R = 2.0$ and (b) $\rho_R = 0.5$. 
Conclusion

◆ Development of EDN 3D interconnection scheme
  – Concurrent shape of luminosity error with errors in each of primaries
  – User-adjustable emphasis of weighted error in luminosity
  – Appearance of smoother and more homogeneous image

◆ Future work
  – Design of filters to match a more precise model of the HVS
Floyd-Steinberg

\[
\begin{array}{ccc}
\cdot & 7 \\
3 & 5 & 1 \\
\end{array}
\times \frac{1}{16}
\]
- Previously developed 2D filter

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\[
\times \frac{1}{96}
\]
Block diagram of Hopfield Network.
Figure 1: Block diagram of recursive error diffusion modulator.
\( X = L X_c \)