Resolution Enhancement by Interpixel Interference Elimination

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Abstract

◆ Proposed method
  – Resolution enhancement algorithm
    • Iterative method
      – Estimation of interpixel interference
      – Elimination of interpixel interference
    • Use of a Gaussian mixture
      – Local image constraints
      – Local variation indicator (LVI)
    • Recovery of high resolution image by estimating and compensating the missing high-frequency details iteratively
Introduction

◆ Super-resolution

- Definition
  • Estimation of the higher-resolution images from lower-resolution images

- Traditional concept
  • Recovery of a single image from multiple low-resolution input image

- Recent concept
  • Incorporation of the work of recovering a higher-resolution image with a single input image
Proposed method

- Use of Gaussian mixture
  - Confining the Gaussian component
    - Use of image derivative priors
  - Use of Local variation indicator (LVI) in Gaussian weight
    - Incorporation of variant image degradation factor
      » PSF, motion blur, and quantization error
    - Procedure of LVI estimation
      » Estimation from continuous local frames
      » Estimation of interpixel interference by Gaussian mixture

- Repeat interference estimation and elimination until the interpixel interference is negligible
Prior Work

- Traditional image super-resolution method
  - Some techniques
    - Concentration of the resampling procedure during image degradation
      - Dense optical flow for estimating the subpixel motion
      - Convex set constraint
  - Some other techniques
    - Concentration of finding the appropriate regularization constraint for the ill-conditioned problem
      - Representation of forward model in frequency and spatial domain
– Pure global translational motion of the camera

\[ I_0(x; t) = I_s(x + \Delta_t) \quad t = 1, \ldots, R \]  

where \( \Delta_t \) describes the translational motion for the \( t \)th observation

\( I_0(x; t) \) is \( t \)th observations

\( I_s \) is source

» Representation in the frequency domain

\[
F \{ \{ I_0(x; t) \} \} = e^{j2\pi(\Delta_t^T w)} F \{ I_s(x) \} 
\]  

(2)
DFT of the shifted and sampled low-resolution images

\[ I_y(w; t) = \frac{1}{T_1 T_2} \sum_{i_1=-\infty}^{+\infty} \sum_{i_2=-\infty}^{+\infty} I_0 \left( \frac{w_1}{N_1 T_1} + \frac{i_1}{T_1}, \frac{w_2}{N_2 T_2} + \frac{i_2}{T_2}; t \right) \]

\[ w = (w_1, w_2) \]  

where \( T_1 \) and \( T_2 \) are sampling intervals

\( N_1 \) and \( N_2 \) are magnification factor in each direction

\( I_0(F\{I_0\}) \) is CFT of the scene

\( I_f(w; t) \) is DFT of the shifted and sampled low-resolution image
- Description of spatial domain approaches

\[ L = A_1 \times \cdots \times A_a H + n \]
\[ H = [I_h(t-p), \ldots, I_h(t+p)] \]
\[ L = [I_l(t-p), \ldots, I_l(t+p)] \]

where \( I_h \) is vector by lexicographically ordered high-resolution image
\( I_l \) is the vector by lexicographically ordered low-resolution image
\( A_1 \times \cdots \times A_a \) is separately model the different deterioration procedure

» MAP estimators using the huber edge-penalty function as a priori in bayesian framework

» Use of a robust regularization as an additional constraint
Recent image super-resolution conception

- Difference from interpolation using different kernels
  - Addition to more compatible details
    - Reduction perceived loss
    - Facilitation of the image representation ability

- Use of Markov random field (MRF)
  - Compensation of the high-frequency component learned from the MRF model

- Spatial and temporal constraint
Proposed algorithm

- Recovery of image details by predicting high-frequency loss iteratively from estimate LVI

Fig. 1. The diagram of the proposed algorithm
Algorithm Framework

- Ideal image acquirement
  - Spatial and temporal direction

\[ I = S(x, y; t) * \delta(x - x_i, y - y_i) * \delta(t - t_0) \]  

where \( t_0 \) is the 0th image is located at \((x_i, y_i)\)

\( S \) is the 2D continuous function describing the scene

\[ \delta(x - x_i, y - y_i) = \begin{cases} 1 & \text{if } x = x_i \text{ and } y = y_i, \text{ and} \\ 0 & \text{otherwise} \end{cases} \]

\[ \delta(t - t_0) = \begin{cases} 1 & \text{if } t = t_0 \\ 0 & \text{otherwise} \end{cases} \]
Practical image acquirement

— low pass procedure in the spatial sampling

\[ I_0(u, v; t_0) = S(x, y; t) ** h_i(x - u, y - v; t_0) \]  \hspace{1cm} (6)

where \((u, v)\) defines pixels on the low-resolution image coordinate system

\[ I_0(x_i, y_i; t_0) = S(x, y; t_0) ** h_i(x - x_i, y - y_i; t_0) \]  \hspace{1cm} (7)

where \((x_i, y_i)\) is used to represent the discrete sampling of \((x, y)\) on high resolution image coordinate system
• Each low-resolution image pixel corresponds to a disjoint high-resolution image neighborhood

Fig. 2. Illustration of the original low-resolution image coordinate system and the high-resolution image coordinate system.
Gaussian mixture model to predict the interference

— Superposition of the interference from neighbor pixels

\[ I_0(x_i, y_i; t_0) = S(x_i, y_i; t_0) + \sum_k f\{i, i_k; t_0\} \quad i_k \in D_i. \quad (8) \]

where \( f\{i, i_k; t_0\} \) is interference on pixel \((x_i, y_i)\) from its neighboring pixel \((x_{i_k}, y_{i_k})\)

• Denotation of neighborhood

\[ D_{i} = \{i_k : d(x_i, x_{i_k}) \leq \zeta, x_i = (x_i, y_i), x_{i_k} = (x_{i_k}, y_{i_k})\} \]
— Denotation of integral interference

\[ F(i; t_0) = \sum_k f \{ i, i_k; t_0 \} \]  

(9)

where \( F(i; t_0) \) is determined by the image derivative prior

— Use of a probability model

• Modeling of probability of integral interference \( F(i; t_0) \) by Gaussian mixture

\[ F(i; t_0) \propto \sum_k w_k N \left( f; \partial I(i, i_k; t_0), \sigma d_{i,i_k} \right) \]  

(10)

\[ \partial I(i, i_k; t_0) = I_0(x_{i_k}, y_{i_k}; t_0) - I_0(x_i, y_i; t_0) \]

where \( w_k \) is the weight for each Gaussian component

\( d_{i,i_k} \) is the Euclidean distance between pixel and its neighboring pixel

\( \partial I(i, i_k; t_0) \) defines image derivative prior between pixel and neighboring pixel
• Gaussian characterization

$$N \left( F; \partial I(i, i_k; t_0), \sigma d_{i,i_k} \right) = \frac{1}{\sqrt{2\pi} \sigma d_{i,i_k}} \times \exp \left\{ -\frac{\left[ F - \partial I(x_{i_k}, y_{i_k}; t_0) \right]^2}{2\sigma^2 d_{i,i_k}^2} \right\} \right.$$ (11)

• Scene estimate

$$\hat{S}(x_i, y_i; t_0) = I_0(x_i, y_i; t_0) - \hat{F}(i; t_0)$$ (12)

where $\hat{S}(x_i, y_i; t_0)$ is the scene estimate
◆ Find the weights from prior knowledge
  — Inclusion of observation error of neighbor pixel in image derivative priors
  — Determination of reliability for each neighboring pixel by determining temporal and spatial variation
  — Introduction of LVI to incorporate temporal and spatial variation
  — Use of LVI to determine the confidence level for the component Gaussians
Temporal variation

— Modeling of the temporal noise of the scene

\[ I_0(x_i, y_i; t_0 + \tau) = S(x_i, y_i; t_0) + n(x_i, y_i; t_0 + \tau) \]

\[ \tau = -p, \ldots, p \]

Where \( n \) is the noise

— Estimation of ML for scene with \( 2p + 1 \) observations

\[ \hat{S}_{ML}(x_i, y_i; t_0) = \frac{1}{2p + 1} \sum_{\tau=-p}^{p} I_0(x_i, y_i; t_0 + \tau) \]

— Definition of temporal variation as bias from the current observation to the ML estimate

\[ b_i(t_0) = I_0(x_i, y_i; t_0) - \hat{S}_{ML}(x_i, y_i; t_0) \]
◆ Spatial variation
   — Assumption
      • unknown and non-uniform point spread function (PSF) to obtain more general for real data
   — Use of a local model for describing the low-pass filtering process
      • Assumption
         – Sufficiently slow local low-pass filtering process
         – Keep of continuous $2p + 1$ frames
— Filtering procedure

\[ \sum_{i} S(x_i, y_i; t_0 + \tau)v(x_i, y_i; t_0) = I_0(u, v; t_0 + \tau) \]  \hspace{1cm} (13)

\[ i \in Q; \quad \tau = -p, ..., p \]

where \( I_0(u, v; t_0 + \tau) \) is the pre-interpolated low-resolution image

\( v(x_i, y_i; t_0) \) are filter coefficients

— Rewriting matrix format

\[ S^T(t_0 + \tau)v(t_0) = I_0(u, v; t_0), \]  \hspace{1cm} (14)

where \( S \) is the matrix whose column vector are lexicographically ordered pixel from \( Q \)

\( I_0 \) is the vector formed by the low-resolution pixel \( (u, v; t_0 + \tau) \) \( \tau = -p, ..., p \)
— Definition of optimal filtering parameter

• Objective function

\[
J(v(t_0)) = \sum_{\tau=-p}^{p} \sum_{i} \left[ K \times I_0(u, v; t_0 - \tau) - v(x_i, y_i; t_0)S(x_i, y_i; t_0 - \tau) \right]^2 + \lambda \Delta v(t_0)
\]  

(15)

Where \( K \) is the number of pixels inside neighborhood \( Q \)

• Smoothing term

\[
s.t.: \|v(t_0)\|_1 = 1.
\]  

(16)

\[
\Delta v(t_0) = \|\partial_x v(t_0)\|_2 + \|\partial_y v(t_0)\|_2 + \|\partial_{xy} v(t_0)\|_2 + \|\partial_{yx} v(t_0)\|_2.
\]  

(17)
• Obtainment of the parameters by a one-step steepest descent

\[
\hat{v}(x_i, y_i; t_0) = v_0 + \mu \sum_{\tau=-p}^{p} \left[ I_0(u, v; t_0 - \tau) - v_0 S(x_i, y_i; t_0 - \tau) \right] S(x_i, y_i; t_0 - \tau)
\]

where \( v(t_0) = v_0 \)

• Normalization

\[
v(x_i, y_i; t_0) = \frac{\hat{v}(x_i, y_i; t_0)}{\|\hat{v}(t_0)\|_1}
\]

\[
\hat{v}(t_0) = [\hat{v}(x_i, y_i; t_0)]_i
\]

• Update

\[
v(x_i, y_i; t_0) \leftarrow v(x_i, y_i; t_0) - v_0
\]
Overall algorithm

- Modeling of LVI as a function $g(\square)$

$$w(i; t_0) = g\left\{-\left[b_i(x_i, y_i; t_0)v(x_i, y_i; t_0)\right]^2\right\}$$

- Use of exponential function

$$w(i; t_0) = \exp\left\{-\left[b_i(x_i, y_i; t_0)v(x_i, y_i; t_0)\right]^2\right\}$$ (20)

- Use of maximum a posteriori (MAP) criteria to estimate $\hat{F}(i; t_0)$

$$\hat{F}(i; t_0) = \arg \max_F (G(F))$$ (21)

where $G(F) = \sum_k w(i_k; t_0)N(F; \partial I(i, i_k; t_0), \sigma d(i, i_k))$
— Finding of the MAP solution by the steepest descent method

\[ F^{j+1}(i; t_0) = F^j(i; t_0) - \alpha \nabla G \]

\[ \nabla G = \frac{\partial}{\partial F} G(F) \bigg|_{F_j} = \sum_k \frac{w_{ik}}{\sqrt{2\pi\sigma_d^2}} \frac{\partial I(i, i_k) - F^j}{\sigma_d^2 d_{i_k}^2} \times \exp \left\{ -\frac{\left[ F^j - \partial I(i, i_k) \right]^2}{2\sigma_d^2 d_{i_k}^2} \right\} \]  

(22)

— Update of the scene estimate

\[ \hat{S}(x_i, y_i; t_0) = I_0(x_i, y_i; t_0) - \hat{F}(i, t_0) \]  

(23)

— Update of all images

\[ I_0^{(new)}(x_i, y_i; t_0 + \tau) \leftarrow \hat{S}(x_i, y_i; t_0 + \tau) \quad \tau = -p, \ldots, p \]
Example of procedure

Fig. 3. The iterative procedure for high-resolution reconstruct.
More discussion in the frequency domain

— Observation at pixel \((x_i, y_i)\)

\[
I(x_i, y_i; t_0) = (h_l ** S)(x_i, y_i; t_0)
\]  
(24)

— Rewriting in frequency domain

\[
I(w_1, w_2; t_0) = \mathcal{F}(I_{(x_i, y_i; t_0)})
\]

\[
S(w_1, w_2; t_0) = \mathcal{F}(S_{(x_i, y_i; t_0)})
\]

\[
L(w_1, w_2; t_0) = \mathcal{F}(L_{(x_i, y_i; t_0)})
\]

\[
I(w_1, w_2; t_0) = L(w_1, w_2; t_0) \mathcal{F}(w_1, w_2; t_0)
\]  
(25)

Where \(\mathcal{F}(\cdot)\) is the Fourier transform,

\[
H(w_1, w_2; t_0) = I(w_1, w_2; t_0) - L(w_1, w_2; t_0)
\]  

is responding high-pass filter for \(L(w_1, w_2; t_0)\),

and \(I(w_1, w_2; t_0)\) is an all pass filter.

— Error

\[
\varepsilon(w_1, w_2; t_0) = H(w_1, w_2; t_0) \mathcal{F}(w_1, w_2; t_0)
\]  
(26)
— Unknown true scene $S(w_1, w_2; t_0)$

- Prediction of the error by using the current high-resolution estimate
  \[
  \varepsilon(w_1, w_2; t_0) = H(w_1, w_2; t_0)I_0(w_1, w_2; t_0)
  \]  \hspace{1cm} (27)

- Summary of the procedure in the frequency domain
  - Prediction of the high-frequency component, or the function of the interpixel interference
    \[
    \hat{\varepsilon}(w_1, t) = H(w_1, w_2; t_0)I^{old}(w_1, w_2; t_0)
    \]  \hspace{1cm} (28)
  - Refinement of the HR estimates
    \[
    I^{new}(w_1, w_2; t_0) = I^{old}(w_1, w_2; t_0) + \hat{\varepsilon}(w_1, w_2; t_0)
    \]  \hspace{1cm} (29)

\[
I^{new}(w_1, w_2; t_0) = [L(w_1, w_2; t_0) + L(w_1, w_2; t_0)H(w_1, w_2; t_0)] \times S(w_1, w_2; t_0)
\]  \hspace{1cm} (30)
• **1D illustration**

![1D illustration of the refinement procedure](image)

**Fig. 4.** 1D illustration of the refinement procedure

• **The flowchart of the algorithm**

![Flowchart of the algorithm](image)

**Fig. 5.** The flowchart of the algorithm
Experimental Evaluation

◆ Face videos with changes in expression
  — Some result from the super-resolution reconstruction

Fig. 6. Example of the experimental results

— Another result

Fig. 7. More results for sequences from Cohn-Kanade facial expression database
Video with Large Head Motion

Fig. 8. Example from sequence with large head motion
Using linear motion blur filter

Fig. 9. Interpixel interference elimination-based resolution enhancement for motion-blurred color face video
Video from an omnidirectional video camera

— Omnidirectional image

Fig. 10. Example image from the ODVC
— Output from ODVC

![Images showing resolution enhancement examples](image)

Fig. 11. Example of the resolution enhancement
Quantitative Comparison
— Quantitative performance evaluation

Fig. 12. Example of the HR reconstruction
Quantitative performance evaluation

Table 1. The mean of each frame’s PSNR for different algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Interpixel Interference Elimination Algorithm</th>
<th>Borman’s Algorithm</th>
<th>IBP Algorithm with Mean</th>
<th>IBP Algorithm with Median</th>
<th>IBP Algorithm with Median and Bias Detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean PSNR</td>
<td>61.80 dB</td>
<td>61.72 dB</td>
<td>58.78 dB</td>
<td>58.84 dB</td>
<td>59.73 dB</td>
</tr>
</tbody>
</table>

Fig. 13. The PSNR of different algorithms

Fig. 14. Mean MSE over all frames with respect to frequency
Other Example

— Proposed algorithm over text subject to evaluate the performance

Fig. 15. Example for synthetic text subject
— Proposed algorithm over text subject to evaluate

Fig. 16. Example for text subject
— Presentation of different magnification factors

Fig. 17. Example for license plate
Conclusion

◆ Proposed method
  – Reconstruction of high-resolution image
    • Interpixel interference elimination
      – Actual interpretation as high-frequency loss
      – A probability model using Gaussian mixture
        » Confinement by the image derivative priors
        » Local variation indicators
      – By subtracting the estimated interpixel interference iteratively