Image-Dependent Gamut Mapping as Optimization Problem

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Abstract

◆ Image-dependent gamut mapping
  – Consideration of optimization problem
  – Comparing to standard reference gamut mapping algorithms
    • Psycho-visual test
Introduction

◆ Gamut
  – Entirety of colors containing in an image or device

◆ Gamut mapping
  – The adaptation of a color image to device limitations
    • Point-to-point mapping of color points
  – Classifying algorithms
    • Device-to-device technique
    • Image-to-device technique
Purpose of this paper

- Exploration of the potential of image-to-device gamut mapping

Optimization approach

- Combining different gamut mapping concepts
- Practical feasibility for different boundary descriptors
- Providing an evaluation in psycho-visual tests
Preliminaries

- Gamut mapping
  - Categorization of gamut mapping
    - Several dimension
      - Device-to-device
      - Image-to-device
    - Area
      - Global mapping
      - Local mapping
  - Typical gamut mapping algorithm
    - Use of gamut border colors
      - Largely dependent boundary surface
    - Complex shapes
      - Not an easy task
)Mathematical optimization
  – General form of optimization

  minimize \( x \quad f(x) \)
  subject to \( c_i(x) \leq 0, \quad i = 1 \ldots n, \quad \text{for all } x \)

  where \( f : \mathbb{R}^d \to \mathbb{R} \) is called the objective function.
  \( c_i : \mathbb{R}^d \to \mathbb{R} \) are functions that define constraints.
  and \( x \in \mathbb{R}^d \) is the optimization variable.

  • Feasible point
    – A point that satisfying all constrains
• Local optimum

\[ U(x^*) \subset \mathbb{R}^d \text{ such that } f(x^*) < f(x) \text{ for all } x \in U(x^*) \setminus \{x^*\} \]

• Global optimum

\[ f(x^*) < f(x) \text{ for all } x \in \mathbb{R} \]

In general, a mathematical optimization problem neither needs to have a local or a global optimum.

− Kind of using problem
  • Linear program
    − Efficient algorithm
  • Convex quadratic program
    − Convex and quadratic function
Gamut mapping as optimization problem

- **Multicriteria optimization problem**
  - Several objective
    - Detail preservation, hue preservation, gray axis preservation, continuity of the mapping, and a low-image distortion
  - Avoiding artifacts

- **Mathematical optimization problem for gamut mapping**
  - Geometric setup of gamut mapping
    - Image gamut $I$ and the target gamut $T$
      - Finite subsets (point clouds)
• Assumption of working color space
  – Approximately hue preserving
  – Approximately perceptually uniform
    » Equal Euclidean distances in colorspace correspond to equal distance in visual perception
  – OSA-USC, hue-linearized CIELAB, and DIN 6164

• Assumption of operations
  – Computing a continuous shape $\text{SHAPE} (T)$
    » Approximation of target gamut $T$
  – Determining for every point in the working color space
  – Determining the intersection points of a line with $\text{SHAPE} (T)$
– Metric on the color space

- Using the Euclidean metric
- Candidate to model low image distortion

\[
\begin{align*}
\text{minimize}_{f} & \quad \sum_{x \in I} \| x - f(x) \|^2 \\
\text{subject to} & \quad f(x) \in \text{SHAPE}(T) \quad \text{for all } x \in I
\end{align*}
\]

(1)

- Restricting class of functions
  - Optimizing a parameterized family of functions
- Approximately hue preserving
  - Allowing choice of a focal point \( c \) in \( \text{SHAPE}(T) \)
  - Restricting the mapping of any point \( x \in I \) to the ray originating in \( c \)
  - Shooting in the direction of \( x \)

\[
f(x) = c + \lambda_x (x - c), \quad \lambda_x \geq 0
\]

where the ray is parameterized by \( \lambda_x \)
– Assumption of star shaped $\text{SHAPE}(T)$ with respect to $c$
  » Every point $x \in \text{SHAPE}(T)$ and line segment $\overline{xc}$
  » Completely contained in $\text{SHAPE}(T)$
  » Constraint $f(x) \in \text{SHAPE}(T)$ for $x \in I$

$$0 \leq \lambda_x \leq \hat{\lambda}_x$$

where $\hat{\lambda}_x$ is such that $c + \hat{\lambda}_x (x - c)$ is the intersection of the boundary of $\text{SHAPE}(T)$

– Optimization problem by $\|x - f(x)\|^2 = (1 - \lambda_x)^2 \|x - c\|^2$

$$\min_{\lambda} \sum_{x \in I} (1 - \lambda_x)^2 \|x - c\|^2$$

subject to $\lambda_x - \hat{\lambda}_x \leq 0$ for all $x \in I$
$$-\lambda_x \leq 0$$ for all $x \in I$ (2)
• Adding additional constraints to the optimization problem
  – Detail preservation
    \[ c_1 \|x - y\| \leq \|f(x) - f(y)\| \quad \text{for all pairs } x, y \in I \]
    where \( c_1 > 0 \) is a constant

  – Continuity
    \[ \|f(x) - f(y)\| \leq c_2 \|x - y\| \quad \text{for all pairs } x, y \in I \]
    where \( c_2 > c_1 \) is a constant

  – Gray axis preservation
    » Position of focal point \( c \) with gray axis
    » Mapping gray axis points onto gray axis points
• Combining everything

\[ \min_{\lambda} \sum_{x \in I} (1 - \lambda_x)^2 \|x - c\|^2 \]

subject to

\[ \lambda_x - \hat{\lambda}_x \leq 0 \text{ for all } x \in I \]
\[ -\lambda_x \leq 0 \text{ for all } x \in I \]
\[ c_1 \|x - y\| - \|f(x) - f(y)\| \leq 0 \text{ for all } x, y \in I, x \neq y \]
\[ \|f(x) - f(y)\| - c_2 \|x - y\| \leq 0 \text{ for all } x, y \in I, x \neq y \]  \( (3) \)
Turning the optimization problem feasible

◆ Theoretical considerations
  – Derivation of theoretical considerations seems infeasible to solve for typical image and target gamut
    • Optimization problem
      – General nonconvex form
        » Constraints are nonconvex
    • Many variables
    • Very large number of constraints
  – Modification of optimization problem
    • Making feasible also in practice
**Number of variables**

- Reduction of the number of variables
  - Discarding all points that do not belong to the boundary of the image gamut
  - Operator \( \text{BOUNDARY}(I) \)
    - Considering to be located on the boundary of the shape
      » Underlies the point set \( I \)
  
  \[
  f : x \mapsto c + \lambda_x (x - c)
  \]

- Mapping form the boundary points to the interior points
  - \( x \notin \text{BOUNDARY}(I) \)
  - Determining \( y \in \text{BOUNDARY}(I) \)
    » Closest point to \( x \)
    » Using angle between two rays
- Projection the point $x$ onto $c_y$
  - If $x'$ is not contained in the segment $c_y$
    \[ f : x' \mapsto f(y) = c + \lambda_y (y - c) \]
    \[ \lambda_x = \frac{\|y - c\|}{\|x' - c\|} \]
- Displacement of $x'$
  \[ x' \mapsto g_y(x') \]
  where $g_y$ is another parameter; examples include linear and nonlinear compression.
- Definition of $\lambda_x$
  \[ \left| c g_y(x') \right| / |cx'| \]
- Final displacement of $x$
  \[ f : x \mapsto c + \lambda_x (x - c) \]

**Fig. 1.** Extending the mapping from boundary points to interior points.
◆ Number of constraints
  – Reduction of the number of constraints
    • Retaining the detail and continuity constrains
      – Points $x, y \in \text{BOUNDARY}(I)$
    • Measuring closeness by the angle $\alpha$
      – Between the rays shooting from the focal point $c$ to $x$ and $y$

◆ Linearization
  – Linearization of the continuity and detail constraints
    • Measuring distances along the rays shooting from $c$ to $x$ and $y$
    • Assumption
      – $y$ is closer to $c$ in Euclidean distance than $x$ $\|x - c\| \geq \|y - c\|$
        
        $\|x - y\|$ by $\|x - c\| - \|y - c\|$
        
        $\|f(x) - f(y)\|$ by $\|f(x) - c\| - \|f(y) - c\|$
• Modification of continuity and detail constraints

\[ c_1 \left( \| x - c \| - \| y - c \| \right) \leq \| f(x) - c \| - \| f(y) - c \| \leq c_2 \left( \| x - c \| - \| y - c \| \right) \]

• Monotonicity constraint

\[ \| x - c \| \geq \| y - c \| \Rightarrow \| f(x) - c \| \geq \| f(y) - c \| \]

• Using \( \| f(x) - c \| - \| f(y) - c \| = \lambda_x \| x - c \| - \lambda_y \| y - c \| \)
  – Modification

\[ c_1 \left( \| x - c \| - \| y - c \| \right) \leq \lambda_x \| x - c \| - \lambda_y \| y - c \| \leq c_2 \left( \| x - c \| - \| y - c \| \right) \]

\[ (c_1 - \lambda_x)\| x - c \| - (c_1 - \lambda_y)\| y - c \| \leq 0 \quad \text{and} \quad (c_2 - \lambda_y)\| y - c \| - (c_2 - \lambda_x)\| x - c \| \leq 0 \]
Combining all the modifications

\[ \min_{\lambda} \sum_{x \in \text{BOUNDARY}(I)} (1 - \lambda_x)^2 \|x - c\|^2 \]

subject to

\[ (c_1 - \lambda_x)\|x - c\| - (c_1 - \lambda_y)\|y - c\| \leq 0 \text{ for all } x, y \in \text{BOUNDARY}(I) \]

\[ \|x - c\| \geq \|y - c\| \]

\[ \angle(c - x, c - y) < \alpha \text{ and } x \neq y \]

\[ (c_2 - \lambda_y)\|y - c\| - (c_2 - \lambda_x)\|x - c\| \leq 0 \text{ for all } x, y \in \text{BOUNDARY}(I) \]

\[ \|x - c\| \geq \|y - c\| \]

\[ \angle(c - x, c - y) < \alpha \text{ and } x \neq y \]

\[ \lambda_x - \hat{\lambda}_x \leq 0 \text{ for all } x, y \in \text{BOUNDARY}(I) \]

\[ -\lambda_x \leq 0 \text{ for all } x, y \in \text{BOUNDARY}(I) \]
**Parameterizations**

- **BOUNDARY\((I)\)**
  - Two different shapes
    - Convex hull
      \[
      \text{conv}\,(I) := \left\{ \sum_{p \in I} \alpha_p \, p \mid \alpha_p \geq 0 \text{ for all } p \in I \text{ and } \sum_{p \in I} \alpha_p = 1 \right\}
      \]
    - Star shape
      \[
      \text{star}_c\,(I) = \left\{ \alpha \, p + (1 - \alpha) \, c \mid p \in I, 0 \leq \alpha \leq 1 \right\}
      \]

*Fig. 2.* Modified line segments in the definition of the star shape.
Compression functions

- Linear compression
  - $\lambda_x = \lambda_y$ for all points $x$
  
  \[ g_y(x) = c + \lambda_y (x - c) \]

- Nonlinear compression
  
  \[ g_y(x) = c + \left( \frac{w}{q} \left( \frac{\lambda_y \tanh \left( \frac{1}{\lambda_y} \tanh^{-1}(q) \right) \left( q \right) \right) \right) + (1 - \omega) \lambda_y \right) (x - c) \]

where $w = [0, 1]$ is a parameter that controls the nonlinearity.

\[ q = \frac{\|x - c\|}{\|y - c\|}. \]

Fig. 3. Comparison of the nonlinear compression function for different parameters $w$ and the clipping function.
◆ Combinations
  – LOptConv.
    • Using convex hull and linear compression
  – NOptConv.
    • Using convex hull and nonlinear compression
  – LOptStar.
    • Using star shape and linear compression
  – NOptStar.
    • Using star shape and nonlinear compression
Test environment

- Using optimization tool MOSEK with JAVA5
- Graphical user interface
- Image gamut to grid in CIELAB space
- Quantization
  - $L^*$-axis is discretized to 101 equal distant values
  - $a^*$ and $b^*$ axes are both discretized to 256 equal distant value
Results

- Effect of using different source gamut descriptions

Using linear compression in all cases

Fig. 4. (Left) Full device gamut, (middle) convex hull of image gamut, and (right) star shape of image gamut of FRUIT image, each compared to target gamut (dark).
Mapping results

- Improvements
  - Brown color for the basket and in the color of the apple

Fig. 5. (Left) FRUIT image mapped with linear compression using device gamut, (middle) convex hull of the image gamut, and (right) star shape of the image gamut.
Gamut of FRUIT image before and after the mapping

Fig. 6. Mapping of the star shape of the image gamut of the FRUIT image into the target gamut. Note that the shape of the image gamut is partially retained due to the detail preservation and continuity constraints. $w$ and the clipping function.
Psycho-visual tests

- Reference gamut mapping algorithms
  - SGCK
    - Combination of GCUSP and the sigmoid lightness mapping and cusp knee scaling
  - Hue preserving minimal $\Delta E$ (HPMinDE)
    - Keeping colors inside the target gamut unchanged
    - Mapping outside colors to the target gamut border
    - Minimum distance clipping with in planes of constant hue
  - Ldev: Linear compression for the device gamut
  - Ndev: Nonlinear compression for the device gamut
Experimental setup

- Using calibrated LCD screen (EIZO cg220 and cg210)
- Test population
  - Student or staff of total 42 persons
  - Total of 4368 paired comparisons
  - One of the persons: 80-160 comparisons on 10-18 different images
- Test images (Constant height 10.5cm)
  - Recommended by CIE
  - Set of 62 images from a newspaper agency

Fig. 7. ISO test images plus the SKI image as recommended by CIE. The captions of the images are used as reference in the paper.
Data analysis

– Obtaining frequency matrix $P$

Table 2. Frequency matrix for the combined test set. The numbers have to be read as the relative number of observations where the row algorithm was preferred over the column algorithm.

<table>
<thead>
<tr>
<th></th>
<th>HPMinDE</th>
<th>SGCK</th>
<th>LDev</th>
<th>LOptConv</th>
<th>LOptStar</th>
<th>NDev</th>
<th>NOptConv</th>
<th>NOptStar</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPMinDE</td>
<td>-</td>
<td>0.53</td>
<td>0.69</td>
<td>0.56</td>
<td>0.64</td>
<td>0.62</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>SGCK</td>
<td>0.47</td>
<td>-</td>
<td>0.30</td>
<td>0.45</td>
<td>0.62</td>
<td>0.66</td>
<td>0.68</td>
<td>0.79</td>
</tr>
<tr>
<td>LDev</td>
<td>0.31</td>
<td>0.70</td>
<td>-</td>
<td>0.72</td>
<td>0.89</td>
<td>0.84</td>
<td>0.78</td>
<td>0.88</td>
</tr>
<tr>
<td>LOptConv</td>
<td>0.44</td>
<td>0.55</td>
<td>0.28</td>
<td>-</td>
<td>0.76</td>
<td>0.66</td>
<td>0.78</td>
<td>0.79</td>
</tr>
<tr>
<td>LOptStar</td>
<td>0.36</td>
<td>0.38</td>
<td>0.11</td>
<td>0.24</td>
<td>-</td>
<td>0.37</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>NDev</td>
<td>0.38</td>
<td>0.34</td>
<td>0.16</td>
<td>0.34</td>
<td>0.63</td>
<td>-</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>NOptConv</td>
<td>0.18</td>
<td>0.32</td>
<td>0.22</td>
<td>0.22</td>
<td>0.28</td>
<td>0.35</td>
<td>-</td>
<td>0.58</td>
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<tr>
<td>NOptStar</td>
<td>0.15</td>
<td>0.21</td>
<td>0.12</td>
<td>0.21</td>
<td>0.30</td>
<td>0.34</td>
<td>0.42</td>
<td>-</td>
</tr>
</tbody>
</table>

– Obtaining $z$-score matrix $Z$

$\gamma = 0.63, N = 140$

$$z_{ij} = \gamma \ln \left( \frac{Np_{ij} + 0.5}{N(1 - p_{ij}) - 0.5} \right)$$

where $N$ is the number of observations for one pair of algorithms
$\gamma$ is a scaling constant determined from a linear regression of a linear scale and the accumulated normal distribution.
– Obtaining accuracy score for mapping algorithm

\[ a_j = \sum_{i=1}^{n} \frac{z_{ij}}{n} \]

where \( a_j \) is accuracy score for mapping algorithm \( j \)

Fig. 8. Accuracy scores: Algorithms using device gamuts are in light color, algorithms using the convex hull of the image gamut are in gray color, and algorithms using the star shape of the image gamut are in dark color.
◆ Discussion

– Performance of LDev and HPMinDE
  • Poorer than SGCK and NDev

– Using device gamut with nonlinear compression
  • Better than use of image gamut with linear compression

– Using the star shape instead of the convex hull
  • Obtaining high gain in accuracy score for linear and nonlinear compression
Conclusion

- Image-dependent gamut mapping algorithm
  - Optimization-based
  - Determination of gamut boundary for practical feasibility
  - Results
    - Show of good mapping results
    - Avoiding visual artifacts