Design Considerations Between Color Gamut and Brightness for Multi-Primary Color Displays

Journal of Display Technology
vol. 3, no. 1, Mar. 2007
Mang Ou-Yang and Shih-Wei Huang

Presented by Soo-Jin Sung

School of Electrical Engineering and Computer Science
Kyungpook National Univ.
Key issue for display manufacturers
- How to obtain the maximal brightness under an assigned color point or color gamut

Proposal
- A theory to analyze the relation between brightness and color gamut based on the multi-primary color displays
- Providing a design guideline for optimization between color gamut, color temperature and brightness on the multi-primary color displays
Introduction

- The implementation method for enlarging color gamut
  - Need of accurate reproduction of color images
  - The color gamut
    • Determination by the primary colors of display apparatuses
  - A widened tri-primary color display
    • Expansion by the use of high saturation primary colors
    • Having the color gamut limited within the triangle color gamut
  - A four or higher multiple color display
    • Calling multi-primary color display (MPD)
    • Expansions by means of MPD
    • Easier and cheaper than by widened tri-primary color displays
◆ Restriction of the gamut
  – Decrease of the color gamut when brightness grows up

◆ Proposal
  – Method to project for the boundaries of the color gamut of the multi-primary color display
  – on 2-D chromatic diagram by a polygon region
  – Presenting and overcoming multi-primary color problems on 2-D chromatic diagram
Color mixing theory

- Tri-primary color monochromatic light wavelengths (R · G · B light)
  
  $700, 546.1, \text{ and } 435.8 \text{ nm}$

- Color coordinate $(x, y)$ and tri-stimulus values $X, Y, \text{ and } Z$

\[
\begin{align*}
  x & = \frac{X}{X + Y + Z} , \\
  y & = \frac{Y}{X + Y + Z} , \\
  z & = \frac{Z}{X + Y + Z}
\end{align*}
\]

(1)

- The color coordinates and tri-stimulus values of the mixing color

\[
\begin{align*}
  & C_1 \rightarrow (x_1, y_1) \rightarrow X_1, Y_1, Z_1 \\
  & C_2 \rightarrow (x_2, y_2) \rightarrow X_2, Y_2, Z_2 \\
  & C_3 \rightarrow (x_3, y_3) \rightarrow X_3, Y_3, Z_3 \\
  \end{align*}
\]

\[
\begin{align*}
  X_3 & = X_1 + X_2 , \\
  Y_3 & = Y_1 + Y_2 , \\
  Z_3 & = Z_1 + Z_2
\end{align*}
\]

(2)
Luminous flux of $C_1$ and $C_2$

- Supposition as $Y_1$ and $Y_2$ brightness

The mixing color coordinate $(x_3, y_3)$ of $C_3$

$$x_3 = \frac{X_3}{X_3 + Y_3 + Z_3} = \frac{X_1 + X_2}{(X_1 + Y_1 + Z_1) + (X_2 + Y_2 + Z_2)}$$

$$= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_3 = \frac{Y_3}{X_3 + Y_3 + Z_3} = \frac{Y_1 + Y_2}{(X_1 + Y_1 + Z_1) + (X_2 + Y_2 + Z_2)}$$

$$= \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

where $m_1$ and $m_2$ represent $(Y_1 / y_1)$ and $(Y_1 / y_1)$, respectively.

Mixing color coordinate $(x_3, y_3)$ of $C_3$ is at the center-of-gravity position of weights $m_1 (Y_1 / y_1)$ at $C_1$ and $m_2 (Y_2 / y_2)$ at $C_2$.
Color Gamut on Tri-Primary Color Displays

- The tri-primary color coordinate of red, green, and blue
  \[(x_r, y_r), (x_g, y_g), (x_b, y_b)\]

- The maximal luminous flux of red, green, and blue
  \[Y_{r,\text{max}}, Y_{g,\text{max}}, \text{and } Y_{b,\text{max}}\]
  \[Y_{g,\text{max}} > Y_{r,\text{max}} + Y_{b,\text{max}} > Y_{r,\text{max}} > Y_{b,\text{max}}\]

- Total luminous flux of projector light source, \(Y_o\)
  \[Y_o = Y_{r,\text{max}} + Y_{g,\text{max}} + Y_{b,\text{max}}\]

- Total luminous flux after reduction
  \[Y' = Y'_r + Y'_g + Y'_b\]
– Color gamut boundary

• Discussion with different brightness zone of the possible primary color maximal brightness combination

• The number of all possible combination of the maximal luminous flux of n-primary color

\[ \sum_{m=1}^{n} C(n, m) \]

where \( C(n, m) = n!/[m!(n-m)!] \),
\[ n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1, \]
m! is the same meaning with \( m! \).

• The number of zone that the different brightness combination

\[ C(3, 1) + C(3, 2) + C(3, 3) = 7 \]
• Definition of seven zones

H – zone \( \{ Y_{b,\text{max}} > Y' > 0 \} \), I – zone \( \{ Y_{r,\text{max}} > Y' > Y_{b,\text{max}} \} \),

J – zone \( \{ Y_{r,\text{max}} + Y_{b,\text{max}} > Y' > Y_{r,\text{max}} \} \),

K – zone \( \{ Y_{g,\text{max}} > Y' > Y_{r,\text{max}} + Y_{b,\text{max}} \} \),

L – zone \( \{ Y_{g,\text{max}} + Y_{b,\text{max}} > Y' > Y_{g,\text{max}} \} \),

M – zone \( \{ Y_{r,\text{max}} + Y_{g,\text{max}} > Y' > Y_{g,\text{max}} + Y_{b,\text{max}} \} \), and

N – zone \( \{ Y_{o} > Y' > Y_{r,\text{max}} + Y_{g,\text{max}} \} \).

\[ Y_{g,\text{max}} + Y_{r,\text{max}} + Y_{b,\text{max}} > Y' > Y_{g,\text{max}} + Y_{r,\text{max}} + Y_{b,\text{max}} > Y_{g,\text{max}} + Y_{b,\text{max}} > Y_{r,\text{max}} + Y_{b,\text{max}} > Y_{r,\text{max}} > Y_{b,\text{max}} \]

**Fig. 1.** Seven typical color regions in corresponding brightness.
- Determining the color gamut boundary
  - Choice of total luminous flux $Y'$
  - Determination which zone defined above is $Y'$ belonging to
  - Determination of the mixing color coordinate of color gamut boundary apexes by (1)
  - The region determined by linking these apexes from the last step

- Examples I
  - In case of choice $Y'$ belonging to J-zone
    \[
    \left\{ Y_{r,\text{max}} + Y_{b,\text{max}} > Y' > Y_{r,\text{max}} \right\}
    \]
    \[
    Y_g = Y' \quad \Rightarrow \quad J_1 (x_g, y_g)
    \]
    \[
    Y'_r = Y_{r,\text{max}}, \quad Y'_g = Y' - Y_{r,\text{max}} \quad \Rightarrow \quad J_2
    \]
    \[
    Y'_r = Y_{r,\text{max}}, \quad Y'_b = Y' - Y_{b,\text{max}} \quad \Rightarrow \quad J_3
    \]
    \[
    Y'_r = Y_{b,\text{max}}, \quad Y'_r = Y' - Y_{r,\text{max}} \quad \Rightarrow \quad J_4
    \]
    \[
    Y'_r = Y_{b,\text{max}}, \quad Y'_g = Y' - Y_{b,\text{max}} \quad \Rightarrow \quad J_5
    \]
- Examples II

  - In case of choice \( Y' \) belonging to \( L \)-zone

\[
\{ Y'_{g,\max} + Y'_{b,\max} > Y' > Y_{g,\max} \}
\]

\[
Y'_{g} = Y'_{g,\max}, \quad Y'_{r} = Y' - Y_{g,\max} \quad \Rightarrow \quad L_1
\]

\[
Y'_{r} = Y'_{r,\max}, \quad Y'_{g} = Y' - Y_{r,\max} \quad \Rightarrow \quad L_2
\]

\[
Y'_{r} = Y'_{r,\max}, \quad Y'_{b} = Y'_{b,\max}, \quad Y' = Y' - (Y'_{r,\max} + Y'_{b,\max}) \quad \Rightarrow \quad L_3
\]

\[
Y'_{b} = Y'_{b,\max}, \quad Y'_{g} = Y' - Y'_{b,\max} \quad \Rightarrow \quad L_4
\]

\[
Y'_{g} = Y'_{g,\max}, \quad Y'_{b} = Y' - Y_{g,\max} \quad \Rightarrow \quad L_5
\]

- The color gamut boundary of total luminous flux \( Y' \) in \( L \)-zone
  - Inclusion in the region determined by linking five apexes

\((L_1, L_2, L_3, L_4, \text{and} \ L_5)\)
Examples III

- In case of choice \( Y' \) belonging to \( N \)-zone

\[
\{ Y_0 > Y' > Y_{r,\text{max}} + Y_{g,\text{max}} \}
\]

\[
Y'_r = Y_{r,\text{max}}, \quad Y'_g = Y_{g,\text{max}}, \quad Y'_b = Y' - (Y_{r,\text{max}} + Y_{g,\text{max}}) \quad \Rightarrow \quad N_1
\]

\[
Y'_r = Y_{r,\text{max}}, \quad Y'_b = Y_{b,\text{max}}, \quad Y'_g = Y' - (Y_{r,\text{max}} + Y_{b,\text{max}}) \quad \Rightarrow \quad N_2
\]

\[
Y'_g = Y_{g,\text{max}}, \quad Y'_b = Y_{b,\text{max}}, \quad Y'_r = Y' - (Y_{g,\text{max}} + Y_{b,\text{max}}) \quad \Rightarrow \quad N_3
\]

- The color gamut boundary of total luminous flux \( Y' \) in \( N \)-zone
  - Inclusion in the region determined by linking three apexes
    \((N_1, N_2, \text{and} \ N_3)\)

- \( Y' = Y_o \quad (\rightarrow Y_{r,\text{max}} + Y_{g,\text{max}} + Y_{b,\text{max}}) \quad \Rightarrow \quad \text{white point} \)
Color Gamut on Multi-primary Color Displays

- The color gamut boundary of $Y'$
  - $\sum_{i=0}^{N-1} C(N,i)$ zones by the different brightness
- Different case from tri-primary color displays system
  - The states of zone number $C(N,i)$ of un-adjacent primary color $2 \leq i \leq N - 2$
- Increase of $Y'$ in case of $Y_{p_i,\text{max}} < Y'$
  - Approaching the two apexes of the color gamut boundary to the two neighbor primary color
- Example I for M-zone shown in Fig. 1 (mixing three colors)
  - Increase of $Y'$ in case of $Y_{b,\text{max}} < Y'$
    - Approaching the two apexes of the color gamut boundary of $M_3$ and $M_4$ to the coordinates of red and blue
    - The center-of-gravity positions of blue and red and blue and green respectively
Example I in Fig. 2(a) (mixing more than three colors)

- Increase of $Y'$ in case of $Y_{C_1,\max} < Y'$
  - Approaching the two apexes of the color gamut boundary of $M_1$ and $M_2$
  - To the primary color coordinates of $C_0$ and $C_2$
  - In other words
    - $M_1$ in case of $Y_1 < Y'_1 < Y_0 + Y_1$
      - The center-of-gravity positions of $C_1$ and $C_0$
    - $M_2$ in case of $Y_1 < Y'_1 < Y_1 + Y_2$
      - The center-of-gravity positions of $C_1$ and $C_2$
  - The math form

$$B[M_1, M_2] = B[P(C_1, C_0), P(C_1, C_2)]$$

Represents the color gamut boundary of the line linked by the two apexes, $M_1$ and $M_2$

$P(C_0, C_1)$ is represents the center-of-gravity positions of $C_0$ and $C_1$
– formulas

\[
M_1 = \begin{pmatrix}
\frac{Y_1 x_1 + Y' - Y_1 x_0}{y_1} & \frac{Y_1 y_1 + Y' - Y_1 y_0}{y_0} \\
\frac{Y_1 + Y' - Y_1}{y_1} & \frac{Y_1 + Y' - Y_1}{y_0}
\end{pmatrix}, \quad (4)
\]

\[
M_2 = \begin{pmatrix}
\frac{Y_1 x_1 + Y' - Y_1 x_2}{y_1} & \frac{Y_1 y_1 + Y' - Y_1 y_2}{y_2} \\
\frac{Y_1 + Y' - Y_1}{y_1} & \frac{Y_1 + Y' - Y_1}{y_2}
\end{pmatrix}, \quad (5)
\]

Where \( B[M_1, M_2] \) represents the color gamut boundary of the line linked by the two apexes, \( M_1 \) and \( M_2 \)

\( P(C_0, C_1, C_2, \ldots, C_N) \) represents the center-of-gravity positions of \( C_0, C_1, C_2, \ldots, \) and \( C_N \)
Example II for K-zone shown in Fig. 1 (mixing three colors)

- Increase of $Y'$ in case of $Y_{b,\max}, Y_{r,\max} < Y'$
  - Approaching the two apexes of the color gamut boundary of $M_1$ and $M_2$
    to the coordinates of red and blue

Example II in Fig. 2(b) (mixing more than three colors)

- Increase of $Y'$ in case of $Y_{C_1,\max}, Y_{C_2,\max} < Y'$
  - Approaching the two apexes of the color gamut boundary of $M_1$ and $M_2$
    to the primary color coordinates of $C_0$ and $C_3$
  - In other words
    » $M_1$ in case of $Y_1 + Y_2 < Y' < Y_0 + Y_1 + Y_2$
      The center-of-gravity positions of $C_1, C_2,$ and $C_0$
    » $M_2$ in case of $Y_1 + Y_2 < Y' < Y_1 + Y_2 + Y_3$
      The center-of-gravity positions of $C_1, C_2,$ and $C_3$
  - The math form

$$B[M_1, M_2] = B[P(P(C_1, C_2), C_0), P(P(C_1, C_2), C_3)]$$
$$= B[P(G, C_0), P(G, C_3)]$$
- formulas

$$G = (x_g, y_g) = \left( \frac{Y_1}{y_1} x_1 + \frac{Y_2}{y_2} x_2, \frac{Y_1}{y_1} y_1 + \frac{Y_2}{y_2} y_2 \right)$$

(6)

$$M_1 = \left( \begin{array}{c}
\frac{Y_1 + Y_2}{y_g} x_g + \frac{Y' - Y_1 - Y_2}{y_0} x_0 \\
\frac{Y_1 + Y_2}{y_g} y_g + \frac{Y' - Y_1 - Y_2}{y_0} y_0
\end{array} \right)$$

(7)

$$M_2 = \left( \begin{array}{c}
\frac{Y_1 + Y_2}{y_g} x_g + \frac{Y' - Y_1 - Y_2}{y_3} x_3 \\
\frac{Y_1 + Y_2}{y_g} y_g + \frac{Y' - Y_1 - Y_2}{y_3} y_3
\end{array} \right)$$

(8)
Example III of the three close primary colors in Fig. 2(c)

- Increase of $Y'$ in case of $Y_{C_1,\text{max}}, Y_{C_2,\text{max}}, \text{and } Y_{C_3,\text{max}} < Y'$
  - Approaching the two apexes of the color gamut boundary of $M_1$ and $M_2$
    to the primary color coordinates of $C_0$ and $C_4$
  - In other words
    - $M_1$ in case of $Y_1 + Y_2 + Y_3 < Y' < Y_0 + Y_1 + Y_2 + Y_3$
      The center-of-gravity positions of $C_1, C_2, C_3,$ and $C_0$
    - $M_2$ in case of $Y_1 + Y_2 + Y_3 < Y' < Y_1 + Y_2 + Y_3 + Y_4$
      The center-of-gravity positions of $C_1, C_2, C_3,$ and $C_4$

- The math form

\[
B[M_1, M_2] = B[P(P(C_1, C_2, C_3), C_0), P(P(C_1, C_2, C_3), C_4)] = B[P(G, C_0), P(G, C_4)]
\]

where $G$ represents the center-of-gravity positions of $C_1, C_2,$ and $C_3$. 
- formulas

\[
G(x_g, y_g) = \left( \begin{array}{c}
\frac{Y_1 x_1 + Y_2 x_2 + Y_3 x_3}{y_1} \frac{Y_1 y_1 + Y_2 y_2 + Y_3 y_3}{y_1} \\
\frac{Y_1 y_1 + Y_2 y_2 + Y_3 y_3}{y_1} \frac{Y_1 y_1 + Y_2 y_2 + Y_3 y_3}{y_1}
\end{array} \right)
\]

\[
M_1 = \left( \begin{array}{c}
\frac{Y_1 + Y_2 + Y_3 y_g}{y_g} + \frac{Y' - Y_1 - Y_2 - Y_3}{y_0} x_0 \\
\frac{Y_1 + Y_2 + Y_3 y_g}{y_g} + \frac{Y' - Y_1 - Y_2 - Y_3}{y_0} y_0
\end{array} \right)
\]

\[
M_2 = \left( \begin{array}{c}
\frac{Y_1 + Y_2 + Y_3 y_g}{y_g} + \frac{Y' - Y_1 - Y_2 - Y_3}{y_4} x_0 \\
\frac{Y_1 + Y_2 + Y_3 y_g}{y_g} + \frac{Y' - Y_1 - Y_2 - Y_3}{y_4} y_0
\end{array} \right)
\]
– General presentation

• When \( Y_i + Y_{i+1} + \ldots + Y_j < Y' < Y_{i-1} + Y_i + Y_{i+1} + \ldots + Y_j \)
  \[ Y_i + Y_{i+1} + \ldots + Y_j < Y' < Y_i + Y_{i+1} + \ldots + Y_j + Y_{j+1} \]
• Increase of \( Y' \)
  – Approaching two apexes of the color gamut boundary to the two neighbor primary color nearest the \( j-i+1 \) close primary colors
    where \( j-i+1 \geq 0 \)
  – The shape of the N primary color chromaticites
    » Convex polygon

– The math form

\[
B[M_1, M_2] = B[P(P(C_i, C_{i+1}, \ldots, C_j), C_{i-1}), P(P(C_i, C_{i+1}, \ldots, C_j), C_{j+1})] \\
= B[P(G, C_{i-1}), P(G, C_{j+1})]
\]

where \( G \) represents the center-of-gravity positions of \( C_i, C_{i+1}, \ldots, C_{j-1} \) and \( C_j \).
\[ G = (x_g, y_g) = \left( \frac{\sum_{k=i}^{j} Y_k x_k}{\sum_{k=i}^{j} y_k}, \frac{\sum_{k=i}^{j} Y_k}{\sum_{k=i}^{j} y_k} \right) \]

\[ M_1 = \left( \begin{array}{ccc}
\frac{\sum_{k=i}^{j} Y_k}{y_g} x_g + \frac{\sum_{k=i}^{j} Y_k}{y_{i-1}} x_{i-1} & \frac{\sum_{k=i}^{j} Y_k}{y_g} + \frac{\sum_{k=i}^{j} Y_k}{y_{i-1}}
\frac{\sum_{k=i}^{j} Y_k}{y_g} + \frac{\sum_{k=i}^{j} Y_k}{y_{i-1}} & \frac{\sum_{k=i}^{j} Y_k}{y_g} + \frac{\sum_{k=i}^{j} Y_k}{y_{i-1}}
\end{array} \right) \]

\[ M_2 = \left( \begin{array}{ccc}
\frac{\sum_{k=i}^{j} Y_k}{y_g} x_g + \frac{\sum_{k=i}^{j} Y_k}{y_{i+1}} x_{j+1} & \frac{\sum_{k=i}^{j} Y_k}{y_g} + \frac{\sum_{k=i}^{j} Y_k}{y_{i+1}}
\frac{\sum_{k=i}^{j} Y_k}{y_g} + \frac{\sum_{k=i}^{j} Y_k}{y_{i+1}} & \frac{\sum_{k=i}^{j} Y_k}{y_g} + \frac{\sum_{k=i}^{j} Y_k}{y_{i+1}}
\end{array} \right) \]
– Obtaining the color gamut boundary of multi-primary color display system
– Color gamut boundary
  • The lines linked by these available color gamut boundary apexes

Fig. 2. (2a) nearest one close primary colors, (2b) nearest two close primary, (2c) nearest three close primary.
Simulation and Experiments of Tri-Primary Color Displays

- **Experiment I**
  - Using LCD monitor (LG L1970HR-WF) and a color analyzer (CA-210)
  - Color chromaticity coordinates
    - $(0.6442, 0.3196)$, $(0.2941, 0.6242)$, and $(0.1409, 0.0795)$
  - Maximal luminous flux of red, green, and blue
    - $55.55$, $177.9$, $28.56$

- **Experiment II**
  - Using LCD projector (professional EX-2700 Series) and a color analyzer (Minolta CL-200)
  - Color chromaticity coordinates
    - $(0.6245, 0.3604)$, $(0.3833, 0.5959)$, and $(0.1416, 0.0474)$
  - Maximal luminous flux of red, green, blue, and projection source
    - $214.8$, $2018$, $72$, $2304.8$
Fig. 3. Block diagram of three experimental apparatus. (a) and (b) were using a LCD monitor and a LCD projector for a triprimary color system. (c) are using a LCD projector and a DMP projector for a fourprimary color system.

Fig. 4. (a): A photo of the equipment of type (a) and (b) in Fig. 3 is shown. (b): A photo of the equipment of type (c) in Fig. 3 is shown.
Results

• Experiment I
  
  - Representation of the two individual J (Y' = 80) and L (Y' = 185) regions of tri-primary color system (Fig. 5(a))
  
  - Representation of the two individual K (Y' = 1600) and J (Y' = 280) regions of tri-primary color system (Fig. 5(b))

Fig. 5. (a) Experimental results of the LCD monitor for tri-primary color system compare with the color gamut boundary of 185 and 80 nit. (b) Experimental results of the LCD projector for tri-primary color system compare with the color gamut boundary of 1600 and 280 lx.
Simulation and Experiment of Four-Primary Color Displays

- Using two projectors with the different tri-primary colors to simulate the four-primary color display
  - Projection as the three colors of red, green, and blue
  - Projection as the fourth color of yellow
- The four-primary color chromaticity coordinates
  - \((0.5948, 0.3627), (0.3249, 0.5727), (0.1514, 0.0989), \) and \((0.4055, 0.5718)\)
- Maximal luminous flux of red, green, and yellow
  - 569, 2472, 334, and 7135
- Results
  - \(X, y,\) and coordinate axes
    - The coordinate axes of the CIE 1931 and total luminous flux
  - The color gamut with higher luminous flux is small than the ones with lower luminous flux in CIE 1931 color coordinate \((x, y)\)

Fig. 6. (a) 3-D contour of the gamut boundary of four primary colors under different brightness. (b) Experimental result of the two projectors for four-primary color system compare with the color gamut boundary of 9000 and 5500 lx
Maximal Brightness on an Assigned Color Point

- Maximal Brightness on Tri-Primary Color Displays
  - Chromaticity coordinates of tri-primary colors, R, G, and B
    \[ (x_R, y_R), (x_G, y_G), \text{ and } (x_B, y_B) \]
  - Maximal value of three primary color luminous flux, \( Y_R, Y_G, \text{ and } Y_B \)
    \[ Y_{R,MAX}, Y_{G,MAX}, \text{ and } Y_{B,MAX} \]

Fig. 7. 2-D diagram of the gamut boundary of tri-primary colors is divided into three zones. And the luminous flux combination of three zones is shown.
– The mixing chromaticity coordinates of three colors

\[
\begin{align*}
x_{\text{mix}} &= \frac{m_R x_R + m_G x_G + m_B x_B}{m_R + m_G + m_B} \\
y_{\text{mix}} &= \frac{m_R y_R + m_G y_G + m_B y_B}{m_R + m_G + m_B}
\end{align*}
\]

where \( m_R = \frac{Y_R}{y_R} \), \( m_G = \frac{Y_G}{y_G} \), and \( m_B = \frac{Y_B}{y_B} \)

• Determining one of three variables in the three primary color luminous flux combination, \( Y_R \), \( Y_G \), and \( Y_B \)
– For example, the solution of the two variables \((Y_R = Y_{R,\text{Max}})\)

\[
Y_G = \frac{x_{\text{mix}} - x_B}{y_B} \left[ Y_R \times y_{\text{mix}} - y_R \right] - \frac{y_{\text{mix}} - y_B}{y_B} \left[ Y_R \times x_{\text{mix}} - x_R \right] \]

\[
Y_B = \frac{x_{\text{mix}} - x_G}{y_G} \left[ Y_R \times y_{\text{mix}} - y_R \right] - \frac{y_{\text{mix}} - y_G}{y_G} \left[ Y_R \times x_{\text{mix}} - x_R \right] \]

– Obtaining the right combination of the three primary color luminous flux, \(Y_R, Y_G,\) and \(Y_B\)

  • When the displays color temperature transformation is performed under least luminous flux loss
Maximal Brightness on Multi-primary Color Displays

- Chromaticity coordinates of four-primary colors, $C_1, C_2, C_3,$ and $C_4$

$$\Rightarrow (x_{C_1}, y_{C_1}), (x_{C_2}, y_{C_2}), (x_{C_3}, y_{C_3}), \text{ and } (x_{C_4}, y_{C_4})$$

- Maximal value of four primary color luminous flux, $Y_{C_1}, Y_{C_2}, Y_{C_3}, \text{ and } Y_{C_4}$

$$\Rightarrow Y_{C_1,max}, Y_{C_2,max}, Y_{C_3,max}, \text{ and } Y_{C_4,max}$$

- Having eight zones for the four-primary color gamut

Fig. 8. 2-D diagram of the gamut boundary of tri-primary colors is divided into eight zones. And the luminous flux combination of eight zones is shown.
- Having only two variables in the four primary color luminous flux combinations

- Example I
  - In case of \( Y_{C_1} = Y_{C_1,\max} \) and \( Y_{C_3} = 0 \)
    - Location of the display color temperature chromaticity coordinate on the color gamut boundary in zone 1

- Example II
  - In case of \( Y_{C_1} = Y_{C_1,\max} \) and \( Y_{C_2} = Y_{C_2,\max} \)
    - Location of the display color temperature chromaticity coordinate on the color gamut boundary in zone 5
- Mixing chromaticity coordinates of four color

\[
x_{\text{mix}} = \frac{\sum_{i=1}^{p} m_{C_i} x_{C_i}}{\sum_{i=1}^{p} m_{C_i}}
\]
\[
y_{\text{mix}} = \frac{\sum_{i=1}^{p} m_{C_i} y_{C_i}}{\sum_{i=1}^{p} m_{C_i}}
\]

\[
m_{C_i} = \frac{Y_{C_i}}{y_{C_i}}.
\]

(19) (20) (21)

- P is equal to four
- Determination of the values for two of the four variables in the four primary color luminous flux combination
- Remaining two variables called \( Y_{C_m} \) and \( Y_{C_n} \)
– The solution of the two variables

\[
Y_{C_m} = \frac{\sum_{i=1}^{p} Y_{C_i} f(y_{C_i}) - f(y_{C_m}) \sum_{i=1}^{p} Y_{C_i} f(x_{C_i})}{f(x_{C_m}) f(y_{C_m}) - f(x_{C_m}) f(y_{C_m})}, \quad (22)
\]

\[
Y_{C_n} = \frac{f(x_{C_m}) \sum_{i=1}^{p} Y_{C_i} f(y_{C_i}) - f(y_{C_n}) \sum_{i=1}^{p} Y_{C_i} f(x_{C_i})}{f(x_{C_n}) f(y_{C_n}) - f(x_{C_n}) f(y_{C_n})}, \quad (23)
\]

where

\[
f(x_{C_i}) = \frac{y_{C_m}^\text{mix} - y_{C_i}}{y_{C_i}} \quad \text{and} \quad f(y_{C_i}) = \frac{y_{C_m}^\text{mix} - y_{C_i}}{y_{C_i}}
\]
– Determining the four vertex point of zone area and N primary color luminous flux combination
  - When we perform display color temperature transformation under least luminous flux loss

– Determination of the N-primary color luminous flux combination

\[
Y_{C_k} = Y_{C_{k,\max}}, \quad \text{for} \quad i \leq k \leq i + j
\]

\[
Y_{C_k} = Y_{C_{m,\max}}, \text{ and } Y_{C_{n}}, \quad \text{for} \quad k = i - 1 = m;
\]

\[
k = i + j + 1 = n
\]

\[
Y_{C_k} = 0 \quad \text{others}
\]
Fig. 9. An algorithm diagram for determining the four vertex point of zone area and N primary color luminous flux combination, when we perform display color temperature transformation under least luminous flux loss.
# Color Temperature Design

- **Experiment**
  - Using LCD projector (professional EX-17020 Series) and a color analyzer (Minolta CL-200)

## Table 1. List Data of Chromaticity Coordinates and Luminance Flux

<table>
<thead>
<tr>
<th>Measured from projector</th>
<th>Under subtracting light leakage effect</th>
<th>the luminous flux without light leakage effect after color temperature transformation</th>
<th>the luminous flux with light leakage effect after color temperature transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Red</strong></td>
<td>((x_r, y_r, Y_r) = (0.6495, 0.3367, 634))</td>
<td>((x_{ro}, y_{ro}, Y_{ro}) = (0.6623, 0.3373, 614.5))</td>
<td>(Y_{Ro}=614.5) (lx) (Y_R=634) (lx)</td>
</tr>
<tr>
<td><strong>Green</strong></td>
<td>((x_g, y_g, Y_g) = (0.3505, 0.6221, 4724))</td>
<td>((x_{go}, y_{go}, Y_{go}) = (0.3512, 0.6246, 4704.5))</td>
<td>(Y_{Go}=2380.4) (lx) (Y_G=2399.9) (lx)</td>
</tr>
<tr>
<td><strong>Blue</strong></td>
<td>((x_b, y_b, Y_b) = (0.1395, 0.0505, 255))</td>
<td>((x_{bo}, y_{bo}, Y_{bo}) = (0.1379, 0.0472, 235.5))</td>
<td>(Y_{Bo}=163.2) (lx) (Y_B=182.7) (lx)</td>
</tr>
<tr>
<td><strong>White point</strong></td>
<td>((x_w, y_w, Y_w) = (0.3166, 0.3874, 5568))</td>
<td>((x_{wo}, y_{wo}, Y_{wo}) = (0.3168, 0.3877, 5548.5))</td>
<td></td>
</tr>
</tbody>
</table>
Example

- Color temperature translation of the projector color coordinate to color temperature 5500K (0.3324, 0.3474)
- Luminous flux for each primary
  \[ Y_R = Y_{R,\text{max}}, Y_G = Y_m, \text{ and } Y_B = Y_n \]
- Color temperature transformation without light leakage effect
  \[ Y_{R0} = 614.5, Y_{G0} = 2380.4, \text{ and } Y_{B0} = 163.2 \]
- Set the color pattern signal of the closest correspondence luminous
  - (255, 188, 213)
- Chromaticity coordinates of such color pattern signal level
  - (0.3282, 0.3432)
- Luminous flux, \( Y'_w \)
  - 3160 lx
• Corresponding color temperature
  – 5660 k
• Lose luminous flux of ideal
  – 2396.4 flux

  – The 2396.4 flux is the least luminous flux loss if we perform the projector color temperature at 5500k color temperature
Conclusions

🔹 Proposed method
  – Determining How to lose the least brightness sacrifices
    • When the display color coordinate of color temperature translates to a specific color coordinate

🔹 Results
  – Finding that the color gamut boundary with lower brightness will cover the one with higher brightness
  – Display of certain colors below a certain luminous flux \( Y' \)