A robust algorithm for feature point matching

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Proposed method

- A new feature point matching algorithm
  - Main contributions
    - Choosing a feature response function that is different from the one given by Harris
    - Presenting a new constraint
      » Match strength
    - Extending the traditional assignment algorithm
Introduction

◆ Three stages of algorithm for feature point matching
  – Feature point extraction
    • Using non-linear filter
      – SUSAN corner detector by Smith
    • Based on curvature
      – Kitchen and Rosenfeld’s method
      – Extracting edges in advance
      – Finding out the feature points by using the information of curvature of edges
    • Exploiting the change of pixel intensity
      – Harris and Stephens’ method
– Feature point matching
  • Finding out the pixel pairs projected by the same points of a scene
  • Divided into two categories
    – Area based matching (ABM)
    – Feature based matching (FBM)

– Outlier elimination
  • Elimination of false matching points
Feature point extraction

- Feature response function
  - Similar to Harris method
    - Sensitive to noise
  - Using another feature point response

- Average gradient matrices $M$
  - Calculating at each pixel

\[
M = \begin{bmatrix}
\langle \left( \frac{\partial I}{\partial x} \right)^2, W \rangle & \langle \left( \frac{\partial I}{\partial x} \times \frac{\partial I}{\partial y} \right), W \rangle \\
\langle \left( \frac{\partial I}{\partial x} \times \frac{\partial I}{\partial y} \right), W \rangle & \langle \left( \frac{\partial I}{\partial y} \right)^2, W \rangle
\end{bmatrix}
\]
– Feature response function used by Harris

\[ f_{resp}(x, y) = Det(M) - kTrace^2(M) \]

where \( k = 0.04 \) default value proposed by many papers

◆ Using feature response function in this paper

\[ f_{resp}(x, y) = \min(\lambda_1(x, y), \lambda_2(x, y)) \]  \hspace{1cm} (1)

where \( \lambda_1, \lambda_2 \) eigenvalues of \( M \)

– Average ratio of change in intensity
  • Large eigenvalues
    – Causing an important change of gray level in a small motion
– Based on analysis
  • Using the smaller one of eigenvalues
  • Using the threshold to suppress false feature point
    – Choosing by the histogram of small eigenvalue
– Effective noise suppression

Fig. 1. Comparison of the algorithm for feature point extraction: (a) our algorithm and (b) Harris’ algorithm with default k=0.04.
Feature point matching

◆ Match strength

– Local area correlation coefficient (LACC)

\[
c_{ij} = \frac{\sum_{k=-n}^{n} \sum_{l=-m}^{m} [I_1(u_i^1 + k, v_i^1 + l) - \bar{I}_1(u_i^1, v_i^1)] \times \sum_{j=-n}^{n} \sum_{l=-m}^{m} [I_2(u_j^2 + k, v_j^2 + l) - \bar{I}_2(u_j^2, v_j^2)]}{(2n+1)(2m+1) \sqrt{\sigma_i^2(I_1) \times \sigma_j^2(I_2)}}
\]

where \( I_1, I_2 \) intensities of the two images
\((u_i^1, v_i^1), (u_j^2, v_j^2) \) ith and jth feature points
\( m, n \) half-width and half-length of the sliding window

◆ Average intensity of the window

\[
\bar{I}(u, v) = \frac{\sum_{i=-n}^{n} \sum_{j=-m}^{m} I(u + i, v + j)}{(2m+1)(2n+1)}
\]
• Standard variance of the window

\[ \sigma = \sqrt{\sum_{i=-n}^{n} \sum_{j=-m}^{m} I^2(u+i, v+j) \left( \frac{(2m+1)(2n+1)}{(2m+1)(2n+1)} - I^2(u, v) \right)} \]

- Similarity \[ c_{ij} = -1 \sim 1 \]
  - All the \( c_{ij} \) are thresholded
  - LACC > threshold \( \rightarrow \) candidate match
  - LACC < threshold \( \rightarrow \) 0
– Extra constraint for matching
  • Because of outliers
  • Constraint of disparity
    – Setting a radial and setting the LACC of which the distance between two feature points is larger than the radial to 0
  • Support to match

\[ (d_{ij}^2) \]

(Image1) (Image2)

Fig. 2. Similarity measure of relative location of two possibly matched feature point pairs.
– If \( m_1^i \) and \( m_k^i \) match \( m_j^2 \) and \( m_l^2 \), and \( m_1^i \) is close to \( m_k^i \)
  » The relative position of \( m_j^2 \) to \( m_l^2 \) is similar
– To quantify this, defined \( d_{pq}^r = m_p^r - m_q^r \)
  » \( d_{ik}^1 \) is similar to \( d_{jl}^2 \)
– Measure for the similarity

\[
\xi_{ijkl} = \frac{d_{ik}^1 \cdot d_{jl}^2}{\max(\|d_{ik}^1\|, \|d_{jl}^2\|)^2} \tag{3}
\]

– A function with respect to the distance

\[
df_{ik} = \frac{1}{1 + \|m_i - m_k\|^2} \tag{4}
\]
– The percentage of the opposites of a candidate → \( r_n \)

» Consideration of negative \( \xi_{ijkl} \)

– Definition of the match strength

• Symmetry

• All \( ms_{ij} \) are thresholded

\[
ms_{ij} = c_{ij} \left( \sum_{m_k \in \mu_1(m_i)} \max_{m_l \in \mu_2(m_j)} \{c_{kl} \xi_{ijkl} df_{ik}\} + \sum_{m_l \in \mu_2(m_j)} \max_{m_k \in \mu_1(m_i)} \{c_{kl} \xi_{ijkl} df_{jl}\} \right)(1 - r_n) \quad (5)
\]

where \( \mu_1(m_i), \mu_2(m_j) \) the neighboring areas
Fig. 3. Local continuity condition in feature point matching.
Disambiguation

- Even though the match strengths are thresholded, the ambiguity still exists
  - A feature point have more than one candidates
- Solution through optimization
  - One feature point in the first image only matches to a unique one in the second image
    - An assignment problem
    - Classical algorithm for assignment problem
      » Proposed by Kuhn in 1955
      » Requiring coefficient matrix of a square matrix
- Extended assignment problem \( C = (c_{ij})_{m \times n} \)

- **Theorem**
  - Given \( C_e = (c_{ij})_{m \times n} \) is the coefficient matrix of the extended assignment problem
  - Solution is \( Z_e^* = (z_{e,ij}^*)_{m \times n} \)
  - Assuming \( m < n, \ h = n - m \)
  - Constructing a standard assignment problem

\[
C_s = \begin{bmatrix}
(c_{ij})_{m \times n} \\
(a)_{n \times n}
\end{bmatrix}
\]  

(6)

where \( a \) an arbitrary constant

» Solution is \( Z_s^* = (z_{s,ij}^*)_{n \times n} \)

» \( Z_e^* \) is the first \( m \) rows of \( Z_s^* \)
According to the theorem
- Solving the standard assignment problem
- Coefficient matrix is MSE
- First m rows of the solution is the solution of the original problem

\[
MSE = \begin{bmatrix}
\frac{(M - ms_{ij})_{m\times n}}{0_{h\times n}}
\end{bmatrix}_{n\times n}
\] (7)

where \( M \) the maximum of \( ms_{ij} \)
\( h \times n \) an h by n matrix whose entries are all zeros
Elimination of outliers

- Using the LMedS algorithm proposed by Zhang
  - Selecting m samples from the points set
    - Every sample comprises 8 points
    - m determination by the estimated percentage of the outliers
      \[ P = 1 - \left[ 1 - (1 - \varepsilon)^8 \right]^m \]
    - \( \varepsilon \) the estimated proportion of outliers
    - \( P \) the probability of at least one being correct in m samples
  - Calculating the fundamental matrix
    - For sample J use an 8-point algorithm \( F_j, J = 1, \ldots, m \)
– For each $F_j$
  • computing a group of square residuals from the whole set of data
  • Picking out their median $M_j$

$$M_j = \text{med}\left[ \sum_{i=1}^{n} d^2(m_i^2, F_j m_i^1) + d^2(m_i^1, F_j^T m_i^2) \right]$$

where $m_i^1, m_i^2$ the ith pair of matched feature point
d the distance from feature point to line

– Finding the maximum among the $M_j$’s $\rightarrow M_M$
  • Corresponding fundamental matrix $F_M$
– Calculating the robust standard deviation

\[ \hat{\sigma} = 1.4826[1 + 5/(n-8)] \sqrt{M_M} \]

• Assigning a weight to each match

\[ w_i = \begin{cases} 1 & \text{if } r_i^2 \leq (2.5 \hat{\sigma})^2, \\ 0 & \text{otherwise}, \end{cases} \]

\[ r_i = d^2(m_i^2, F_M m_i^1) + d^2(m_i^1, F_M^T m_i^2) \]

– Elimination of outliers having weights of zero
– Optimization problem
  • Using the least square method
  • More accurate estimation of the fundamental matrix

\[
\min \sum_i w_i r_i^2
\]
Experimental results

Fig. 4. Feature point matching of the first scene without elimination of outliers. There are 46 pairs of feature points. The ratio of correct match is 91%.
Fig. 5. The epipolar lines of the first scene.
Fig. 6. Feature point matching of the second scene without elimination of outliers. There are 179 pairs of feature points. The ratio of correct match is 91%.
Fig. 7. The epipolar lines of the second scene.
Fig. 8. Feature point matching of the third scene without elimination of outliers. There are 258 pairs of feature points. The ratio of correct match is 92%.
Fig. 9. The epipolar lines of the third scene.
Fig. 10. Feature point matching of the fourth scene without elimination of outliers. This is a model made with 3D Studio Max. There are 51 pairs of feature points. The ratio of correct match is 91%.
Fig. 11. The epipolar lines of the third scene.
Conclusion and future work

- **Proposed method**
  - A robust algorithm for feature point matching
    - High correct matching ratio

- **Future work**
  - Improving the feature point extraction method
    - According to the theory of human vision
    - Processing each image blockwise