Display gamut comparison with number of discernible colors

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Abstract

◆ The quantitative description of the size of a display gamut
  – Decisive part for designing the wide-gamut display of high saturation primaries

◆ Proposed method
  – Counting the discernible colors in a display gamut with the color-difference formula CIE94

◆ Result
  – Gamut volumes for the three- and four-primary LED displays
    • 1.76 and 1.98 time larger than the standard display
  – Discernible colors of the two LED displays
    • 1.38 and 1.49 time
Introduction

◆ The quantitative description of the size of a display gamut
  – Decisive part for designing the wide-gamut display of high saturation primaries

◆ Method representing the gamut size of a display
  – The area of chromaticity triangle in the 1931 CIE xy diagram
    • Too rough to compare the displays of different primaries
    • Nonuniform color space and no lightness attribute
  – Volume in the 1976 CIELAB color space
    • More uniform than the CIE xyY color space

◆ Number of discernible colors
  – Relating display gamut volume
  – Unrelating the number of displayable color calculated from the bit depth of the RGB signals
Maximum color difference among neighboring signals

\[ \Delta E_{\text{max}} = \text{Max}\{\Delta E(R, G, B; R + 1, G, B), \Delta E(R, G, B; R, G + 1, B), \Delta E(R, G, B; R, G, B + 1)\} \]  

Where \( \text{max}\{\}\) : The maximum function

\( \Delta E(a;b) \) : color difference between the signals of \( a \) and \( b \)

Fig. 1 The maximum color difference statistics for the palette of the display following ITU-R BT.709 and with 24-bit signals
- CIE94 and CIEDE2000 color difference formulas
  - Improve the accuracy of the predicted color difference
  - Based on CIELAB color coordinates
- CIE94
  - Predicting of dependence of the color difference on chroma and hue angle

◆ Proposed method
- Counting the number of discernible colors in a display gamut with CIE94
- Counting the discernible colors with the cylindrical coordinates of lightness, chroma, and hue angle
Presented displays
- ITU-R BT.709 which is called the standard display
- Wide-gamut LED displays
  - Three-primary and four-primary display

**Fig. 2** Chromaticity triangles of the display following ITU-R BT.709 and the wide-gamut LED displays

**Fig. 3** Primary spectra of the considered wide-gamut LED displays
The output CIE tristimulus vector \((X, Y, Z)\) of a three-primary display

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= \begin{bmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{bmatrix}
\begin{bmatrix}
R_L \\
G_L \\
B_L
\end{bmatrix}
\]  
(2)

Where \((X_r, Y_r, Z_r), (X_g, Y_g, Z_g), (X_b, Y_b, Z_b)\) : maximum tristimulus

Vectors of the red, green, and blue primaries

\(R_L, G_L, B_L\): normalized luminance of red, green, and blue primaries

\[0 \leq R_L, G_{YL}, G_L, B_L \leq 1\]

\[
X = \frac{x}{y} Y, \quad Z = \frac{(1-x-y)}{y} Y
\]  
(3)

Y stimulus : relating to the luminance of the primary

X and Z stimulus : deriving from \((x, y)\) color coordinates and the Y stimulus
– Matrix elements of the chromaticity matrix

\[
Y_r + Y_g + Y_b = 1
\]  
\[
\begin{bmatrix}
Y_r \\
Y_g \\
Y_b
\end{bmatrix} =
\begin{bmatrix}
x_r & x_g & x_b \\
y_r & y_g & y_b \\
1 & 1 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
x_w \\
y_w \\
1
\end{bmatrix}
\]  
\[
\begin{bmatrix}
x_r \\
y_r
\end{bmatrix} =
\begin{bmatrix}
x_r \\
y_r
\end{bmatrix} 
\]  
\[
\begin{bmatrix}
z_r \\
z_g \\
z_b
\end{bmatrix} =
\begin{bmatrix}
z_r \\
z_g \\
z_b
\end{bmatrix} 
\]

Where \((x_r, y_r), (x_g, y_g), \text{and} (x_b, y_b)\): the color coordinates of r, g, and b primaries

\[
z_r = 1 - x_r - y_r, \quad z_g = 1 - x_g - y_g, \quad z_b = 1 - x_b - y_b, \quad \text{and} \quad z_w = 1 - x_w - y_w
\]
The output CIE tristimulus vector \((X, Y, Z)\) of a four-primary display

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
X_r & X_y & X_g & X_b \\
Y_r & Y_y & Y_g & Y_b \\
Z_r & Z_y & Z_g & Z_b
\end{bmatrix} \begin{bmatrix}
R_L \\
G_{YL} \\
G_L \\
B_L
\end{bmatrix}
\]  \hfill (2)

Where \((X_y, Y_y, Z_y)\) : maximum tristimulus vectors of the yellowish-green primary

\(G_{YL}\) : normalized luminance of the yellowish-green primaries

\[0 \leq G_{YL} \leq 1\]

\[Y_r + Y_y + Y_g + Y_b = 1\]  \hfill (7)

\[
\begin{bmatrix}
Y_r \\
Y_g \\
Y_b
\end{bmatrix} = \begin{bmatrix}
x_r & x_g & x_b \\
y_r & y_g & y_b \\
z_r & z_g & z_b
\end{bmatrix}^{-1} \begin{bmatrix}
x_w - x_y Y_y \\
y_w - y_y Y_y \\
z_w - z_y Y_y
\end{bmatrix}
\]  \hfill (8)

Where \((x_y, y_y)\) : the color coordinates of the yellowish-green primary

\[z_y = 1 - x_y - y_y\]
The luminance of yellowish-green primary $Y_y$
  - Upper limit of $Y_y$
    - Obtaining by solving Eq. (8) with $Y_g = 0$
  - Maximum $Y_g = 0.793$

**Calculating display gamut boundary**

- Gamut boundary of an n-primary process
  - Founding from the gamut of all of its (n-1)-primary boundary processes in the XYZ color space
- Obtaining the gamut boundary in CIELAB color space
  - Lightness $L^*$ from 1 to 99 in steps of 1
  - Hue angle $h^*$ from 0 to 359 deg in steps of 1 deg
  - Maximum chroma $C_{\text{max}}^* (L_i^*, h_j^*)$
Fig. 4 Gamut cross sections of constant lightness ($L^*$) in CIELAB color space

Fig. 5 Gamut cross sections of constant lightness ($L^*$)
Fig. 6 Gamut cross section of constant lightness ($L^*$) in CIELAB color space

- The gamuts of the LED displays
  - Larger than the standard display
Counting Method

◆ Discernible colors
  – Taking as the grid points within the gamut in CIELAB color space
  – On the plane of constant lightness (L*), the number of unit squares in a*b* coordinates \((\Delta a^* = 1 \text{ and } \Delta b^* = 1)\) are counted

◆ The color-difference formula

\[
\Delta E_{ab}^* = \left( \Delta L^2 + \Delta a^*{}^2 + \Delta b^*{}^2 \right)^{1/2}
\]  

(9)

– Color difference between neighboring grid points
– Call the DeltaE76 in this work
– The number of color points calculated with DeltaE76
  • About the same as the gamut volume
The gamut volume

\[ V = \iiint G(L^*, a^*, b^*) dL^* da^* db^* \]  \hspace{1cm} (10)

Where \( G(L^*, a^*, b^*) \) : The gamut of the display

\[ G(L^*, a^*, b^*) = \begin{cases} 
1, & \text{within gamut} \\
0, & \text{otherwise} 
\end{cases} \] \hspace{1cm} (11)

- Assuming that the volume of the cubic gamut is \( M^3 \) (\( M \) : integer)
- Number of grid points or color points is \( (1 + M)^3 \)
- The ratio of the number of color points or grid volume is \( (1 + 1/M)^3 \)
  - Approaching unit when \( M > 1 \)
The color-difference formula CIE94

\[ C^* = \sqrt{a^*^2 + b^*^2}, \quad h^* = \tan^{-1}\left(\frac{b^*}{a^*}\right) \] (12)

\[ \Delta E_{94} = \left[ \left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C_{ab}^*}{k_c S_C}\right)^2 + \left(\frac{\Delta H_{ab}^*}{k_H S_H}\right)^2 \right]^{1/2} \] (13)

\[ \Delta L^* = L_T^* - L_S^*, \quad \Delta C_{ab}^* = C_T^* - C_S^*, \quad \Delta H_{ab}^* = 2\sqrt{C_T^* C_S^*} \sin\left(\frac{h_T^* - h_S^*}{2}\right) \] (14)

Weighting functions for the chroma

\[ S_L = 1, \quad S_C = 1 + 0.045C_{ab}^*, \quad S_H = 1 + 0.015C_{ab}^* \] (15)

The parameters to adjust the tolerances of the lightness, chroma, and hue angle differences

\[ k_L = k_C = k_H = 1 \]
From the condition

\[ \Delta C^*_a = k_C S_C (C^*_i = 0) \]

- At the concentric rings of constant chroma
- The chroma coordinates of the discernible colors at the i’th hue circle

\[ C^*_i - C^*_{i-1} = k_C \left( 1 + 0.045 \sqrt{C^*_i C^*_{i-1}} \right) \]

where \( i = 1, 2, 3; \quad C^*_0 = 0 \)

From the condition

\[ \Delta H^*_a = k_H S_H \]

- The separation of hue angles for the neighboring discernible colors on the hue circle of \( C^*_i \)

\[ \Delta h^*_i = 2 \sin^{-1} \left( \frac{1 + 0.015 C^*_i}{2 C^*_i} \right) \]
- The number of discernible colors on the hue circle

\[ N_i = \text{Int} \left( \frac{2\pi}{\Delta h^*_i} + 0.5 \right) \]  \hspace{1cm} (18)

Where \( \text{Int}(\bullet) \): The function that truncates the decimal part of its argument

- The separation of hue angles for the neighboring discernible colors on the hue circle of \( C^*_i \)

\[ \Delta h^*_m = \frac{2\pi}{N_i} \]  \hspace{1cm} (19)
Results

- The inaccuracy

\[ \Delta N_i(j) = N_i(j) - 2\pi / \Delta h^*_i(j) \]  

\[ \Delta N_i(j) = N_i(j) - 2\pi / \Delta h^*_i(j) \]  

- The number of discernible colors on the \(i^{th}\) hue circle plane for the \(j^{th}\) constant lightness plane \((L^*_i = j; j = 1, 2, 3, \ldots, 99)\)
  
  - Overestimating \([\Delta N_i(j) > 0]\) or underestimating \([\Delta N_i(j) < 0]\) by less than one
  
  - The summation of \(\Delta N_i(j)\) over all \(i\) and \(j\) indexes
    
    - Standard display \(\Rightarrow 256.5\)
    
    - Three-primary LED display \(\Rightarrow 297.4\)
    
    - Four-primary LED display \(\Rightarrow\) less than 300
      
      - As gamut changes with the luminance of yellowish-green primary \(Y_y\)
      
      - Founging to be less than 300 \((Y_y = 0.793)\)
  
  - Insignification (Eq. 18)
Table 1  Number of discernible colors

<table>
<thead>
<tr>
<th>Gamut size</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ITU-R BT.709 standard display</td>
</tr>
<tr>
<td>Number of CIE94 colors</td>
<td>199,491</td>
</tr>
<tr>
<td>Discernible colors DeltaE76</td>
<td>831,783</td>
</tr>
<tr>
<td>Chromaticity triangle area</td>
<td>0.112</td>
</tr>
<tr>
<td>Gamut volume</td>
<td>831,800</td>
</tr>
</tbody>
</table>
The gamut volume

- Calculation from the boundary descriptor in the cylindrical coordinates by linear approximation

\[
V = \sum_{i=0}^{100} \sum_{j=0}^{359} \frac{1}{2} C_{\text{max}}^* (L_i^*, h_j^*) \Delta L_i \Delta h
\]  

(21)

where \( \Delta h = \pi / 180 \)

\[
\Delta L_i = \begin{cases} 
1. & i = 1,2,\ldots,999 \\
0.5 & i = 0,100 
\end{cases}
\]  

(22)

\[
C_{\text{max}}^* (L_0^*, h_j^*) = \frac{1}{2} C_{\text{max}}^* (L_1^*, h_j^*)
\]

(23)

\[
C_{\text{max}}^* (L_{100}^*, h_j^*) = \frac{1}{2} C_{\text{max}}^* (L_{99}^*, h_j^*)
\]
- Chroma
  - Zero at $L^* = 0$ and 100
  - Only calculable boundary descriptor from $L^* = 1$ and 99
- Gamut volume
  - $L^*$ from 0.5 to 99.5
    - Calculation from the terms of $i = 1$ to 99
  - $L^*$ from 0 to 0.5 and $L^*$ from 99.5 to 100
    - Calculation from the terms of $i = 0$ and 100
- The number of discernible colors counted with DeltaE76
  - The same as the gamut volume as expected (Table 1)
- The number of discernible colors for the standard display counted with CIE94 is 199,491
  - Much less than the 16.7 million of palette colors for 24bit RGB signals
- The same luminances $Y_y$ for the maximum number of discernible colors and gamut volume ($Y_y = 0.28$)
- The chromaticity triangle area and gamut volume
  - 1.82 and 1.98 times larger than the standard display
- The number of discernible colors with CIE94
  - 1.49 times larger than the standard display

$Y_y : 0 \sim 0.793$

Solid line : Color Number
Dash line : Gamut Volume

Fig. 7 Number of discernible colors counted with CIE94 and gamut volume
Flare of display

- Deterioration of the color appearance of a display by the flare that comes from the reflection of room lighting by the screen of the display
- Reduction of the flare by the antireflection coating on the screen

The tristimulus vector of the flare

\[ (X_f, Y_f, Z_f) = \alpha (X_w, Y_w, Z_w) \]

The tristimulus vector of the output of a display

\[ (X_w + X_f, Y_w + Y_f, Z_w + Z_f) \]

by the definition of the CIELAB color coordinates
Fig. 8 The ratio of the number of discernible colors and the ratio gamut volume counted with CIE94 in the presence of the flare and without the flare versus the flare level $\alpha$ for the displays

- Reduce by 30%, with about 3.3 and 2.5% flare levels for the three displays
Figure 9. the number of discernible colors versus the gamut volume for the three displays

- Linear by the shape of dash line as the cased without the flare
- Curved line because the gamut shape of the four-primary display may significantly change with $Y$
Conclusions

◆ Proposed method
  – Counting the number of discernible colors of a display gamut with CIE94
  – Applying for counting the number of discernible colors in the gamuts of other color devices such as printers

◆ The discernible colors
  – Grid points in the cylindrical coordinates of lightness, chroma, and hue angle
  – ITU-R BT.709 standard display
    • 199,491 discernible colors less than the 16.7 million of palette colors for 24-bit RGB signals