Accounting for Inks Interaction in the Yule-Nielsen Spectral Neugebauer Model

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Abstract

- A novel strategy to model dot gain and interaction among inks
  - A printer model based on the Yule-Nielsen spectral Neugebauer equation
  - Designed for a four-ink ink jet printer
Introduction

- Multispectral reproduction
  - Appealing feature of significantly reducing undesirable metamerism effects
  - To produce a color having reflectance equal to that given in input

- Color mixing model of Neugebauer
  - The phenomena
    - The scattering of light and reflections
      - ‘optical dot gain’ or ‘Yule-Nielsen effect’
    - Ink spreading
      - ‘physical’ or ‘mechanical’ dot gain
– Yule-Nielsen
  • Introduction of the n-value for light scattering
  • Advantage: increase of the model’s performance
  • Disadvantage: need of solutions to deal with interactions among inks and of inks with paper

◆ Proposed method
  – A novel method to describe dot gain and ink interaction
    • In the context of the Yule-Nielsen spectral Neugebauer model
    • Analytical models with theory and empiricism
The Yule-Nielsen Spectral Neugebauer Model

The YNSN model for a four ink halftone print

\[
R_{\text{print}}(\lambda) = \left[ \sum_{p=0}^{15} A_p R_p(\lambda)^{1/n(\lambda)} \right]^{n(\lambda)},
\]

Where

- \( R_{\text{print}}(\lambda) \): the reflectance of the printed color
- \( R_p(\lambda) \): the reflectance of the pth Neugebauer primary
- \( n(\lambda) \): the Yule-Nielsen coefficient
- \( A_p \): Neugebauer primary area coverage
◆ Dot placement
   – One of the inks
     • Proportional area coverage to concentration of one ink
   – Two of the inks

Figure 1. Example of dot placement of two inks, yellow and magenta
- Notation of the two inks
  - Area covered by yellow: \( y(1-m) \)
  - Area covered by magenta: \( m(1-y) \)
  - Overlap area: \( my \)

- Neugebauer primary area coverage
  \[
  A_p = \prod_{i=0}^{3} [\text{ind}_{p,i} c_i + (1 - \text{ind}_{p,i})(1 - c_i)],
  \]
  (2)
  Where \( p=0,\ldots,15 \): index of the Neugebauer primary
  \( c=[c,m,y,k] \): vector with the inks concentrations
  \( \text{ind}_{p,i} \): digit of position \( i \) of the index of the primary \( p \)

- Primaries in the order
  white \( \text{ind}_0 = [0,0,0,0] \), \( \ldots \), four inks \( \text{ind}_{15} = [1,1,1,1] \)
### Table 1. The calculus of the area coverage according to the Demichel model

<table>
<thead>
<tr>
<th>Index, p</th>
<th>Neugebauer primary</th>
<th>Area coverage, $A_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>K</td>
<td>$(1-c)(1-m)(1-y)k$</td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>$(1-c)(1-m)y(1-k)$</td>
</tr>
<tr>
<td>3</td>
<td>YK</td>
<td>$(1-c)(1-m)yk$</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>$(1-c)m(1-y)(1-k)$</td>
</tr>
<tr>
<td>5</td>
<td>MK</td>
<td>$(1-c)m(1-y)k$</td>
</tr>
<tr>
<td>6</td>
<td>R</td>
<td>$(1-c)my(1-k)$</td>
</tr>
<tr>
<td>7</td>
<td>RK</td>
<td>$(1-c)myk$</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>$c(1-m)(1-y)(1-k)$</td>
</tr>
<tr>
<td>9</td>
<td>CK</td>
<td>$c(1-m)(1-y)k$</td>
</tr>
<tr>
<td>10</td>
<td>G</td>
<td>$c(1-m)y(1-k)$</td>
</tr>
<tr>
<td>11</td>
<td>GK</td>
<td>$c(1-m)yk$</td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td>$cm(1-y)(1-k)$</td>
</tr>
<tr>
<td>13</td>
<td>BK</td>
<td>$cm(1-y)k$</td>
</tr>
<tr>
<td>14</td>
<td>CMY</td>
<td>$cmy(1-k)$</td>
</tr>
<tr>
<td>15</td>
<td>CMYK</td>
<td>$cmyk$</td>
</tr>
</tbody>
</table>
Modeling Dot Gain and Ink Interaction

◆ Inaccuracy of YNSN model
  – The nonlinear relationship between the theoretical and effective concentration
    • Due to the combination of optical and mechanical dot gain
  – Measurement errors
  – Consider errors in the predictions of the inks area coverage
Figure 2. (a) Plot of the effective concentration of ink against theoretical concentration. The plot has been obtained by computing the effective concentration as

\[
\frac{1}{\Gamma} \sum_{\lambda} \left[ \left( R_{\lambda} - R_{\text{paper,\lambda}} \right) / \left( R_{\text{ink,\lambda}} - R_{\text{paper,\lambda}} \right) \right],
\]

where \( \Gamma \) is the number of lambda samples, for a ramp of 11 cyan color patches printed with on Epson Stylus Color 740, corresponding to theoretical concentrations regularly distributed in the range \([0, 1]\). (b) Plot of the dot gain of ink against theoretical concentration.
Propose

– Example

• Determination of the coverage of the Neugebauer primary $CK$ with theoretical concentration $c_t = [c_t, m_t, y_t, k_t]$
  – Area in the probabilistic model
    \[ c_t (1 - y_t)(1 - m_t)k_t \] (3)
  – Area of the primary
    \[ c_{ck}(c_t)[1 - y_{cyk}(y_t)][1 - m_{cmk}(m_t)]k_{ck}(k_t) \] (4)

– Area of paper coverage

• Computation as the difference between the sum of the area coverage of the inks and their overprints
  \[ A_0 = 1 - \sum_{p=1}^{15} A_p, \quad A_0 \geq 0 \] (5)
Table 2. The calculus of the area coverage from effective concentrations of inks.

<table>
<thead>
<tr>
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<th>Area coverage, $A_p$</th>
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<tbody>
<tr>
<td>1</td>
<td>K</td>
<td>$(1 - c_{ck})(1 - m_{mk})(1 - y_{yk})k_k$</td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>$(1 - c_{cy})(1 - m_{my})y_{y}(1 - k_{yk})$</td>
</tr>
<tr>
<td>3</td>
<td>YK</td>
<td>$(1 - c_{cyk})(1 - m_{myk})y_{yk}k_{yk}$</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>$(1 - c_{cm})m_{m}(1 - y_{my})(1 - k_{mk})$</td>
</tr>
<tr>
<td>5</td>
<td>MK</td>
<td>$(1 - c_{cmk})m_{mk}(1 - y_{myk})k_{mk}$</td>
</tr>
<tr>
<td>6</td>
<td>R</td>
<td>$(1 - c_{cmy})m_{my}y_{my}(1 - k_{cmyk})$</td>
</tr>
<tr>
<td>7</td>
<td>RK</td>
<td>$(1 - c_{cmyk})m_{myk}y_{myk}k_{myk}$</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>$c_c(1 - m_{cm})(1 - y_{cy})(1 - k_{ck})$</td>
</tr>
<tr>
<td>9</td>
<td>CK</td>
<td>$c_{ck}(1 - m_{cmk})(1 - y_{cyk})k_{ck}$</td>
</tr>
<tr>
<td>10</td>
<td>G</td>
<td>$c_{cy}(1 - m_{cmy})y_{cy}(1 - k_{cyk})$</td>
</tr>
<tr>
<td>11</td>
<td>GK</td>
<td>$c_{cyk}(1 - m_{cmyk})y_{cmyk}k_{cyk}$</td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td>$c_{cm}m_{cm}(1 - y_{cmy})(1 - k_{cmk})$</td>
</tr>
<tr>
<td>13</td>
<td>BK</td>
<td>$c_{cmk}m_{cmk}(1 - y_{cmyk})k_{cmk}$</td>
</tr>
<tr>
<td>14</td>
<td>CMY</td>
<td>$c_{cmy}m_{cmy}y_{cmy}(1 - k_{cmyk})$</td>
</tr>
<tr>
<td>15</td>
<td>CMYK</td>
<td>$c_{cmyk}m_{cmyk}y_{cmyk}k_{cmyk}$</td>
</tr>
</tbody>
</table>
– Assume that a drop of ink on the page spreads following the equation

– Linear diffusion equation

\[
\frac{\partial u}{\partial t} = c \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  

(6)

Where \( c \) : a diffusion coefficient

– General solution of the diffusion equation

\[
g(r,t) = u(r) \ast G(r,t) = \int u(q) G(r - q, t) \, dq
\]

(7)

Where

Initial condition at time 0 : \( g(r,0) = u(r) \)

\[
r = \sqrt{x^2 + y^2}
\]

Gaussian kernel : \( G(r,t) = 1/(4\pi ct) \exp[-r^2/(4ct)] \)
– Assuming that the drop has the shape of a Gaussian
– initial ink distribution

\[ u(r) = \frac{1}{\pi} \exp\left(-\frac{r^2}{\sigma^2}\right) \]  \hspace{1cm} (8)

– Parameter related to the amount of ink \( \sigma \)

\[ \frac{1}{\pi} \int \int \exp\left(-\frac{r^2}{\sigma^2}\right) dxdy = \sigma^2 \]  \hspace{1cm} (9)

– Initial condition

\[ g(r,0) = \frac{1}{4\pi c \tau} \exp\left(-\frac{r^2}{4c \tau}\right) \]  \hspace{1cm} (10)

Where \( 3\sigma \) : radius of a circle
\( k = 9\pi\sigma^2 \) : the initial area of the drop
\( \tau = \sigma^2 / 4c \) : parameter
- The area covered by the ink at time \( t \)

\[
a(t) = 36c\pi(t + t) = 9\pi\sigma^2 + 36\pi ct = k + 36\pi ct
\]  

(11)

- The area coverage of the first ink between two inks

\[
a_{1,p} = k_{1,p} + 36\pi ct_{d1} = k_{1,p} + \xi_{1,p}
\]  

(12)

Where subscript \( p \) : the spread on paper

- Spread of a second ink

\[
a_{2,1} = k_{2,p} + 36\pi ct_{d2} + 36\pi c\alpha t_{d2} = k_{2,p} + \xi_{2,p} + \xi_{2,1}
\]  

(13)

Where \( c' = c(1 + \alpha) \) \( k_{2,p} + \xi_{2,p} \): dot gain function of ink on paper

\( k_{2,p} \): the amount of deposited ink

\( \xi_{2,p} \): the dot gain on paper

\( \xi_{2,1} \): the variation of the dot gain due to a previous ink
– The overlap area

\[ A_3 = (k_{1,p} + \xi_{1,p})(k_{2,p} + \xi_{2,p} + \xi_{2,1}) \]  

(14)

Where \( k \) : the quantities of the two inks
\( \xi \) : coefficients, that represent ink spreading
to be determined experimentally

– The areas by all possible combinations of two inks

• Asymmetrical model

\[ A_0 = (1-a_{1,p})(1-a_{2,p}) = 1 - A_1 - A_2 - A_3, \]
\[ A_1 = a_{1,p}(1-a_{2,1}), \]
\[ A_2 = (1-a_{1,p})a_{2,p}, \]
\[ A_3 = a_{1,p}a_{2,1}. \]  

(15)
• Symmetrical model

\[ A_0 = 1 - A_1 - A_2 - A_3, \]
\[ A_1 = a_{1,p} (1 - a_{2,1}), \]
\[ A_2 = (1 - a_{1,2}) a_{2,p}, \]
\[ A_3 = a_{1,2} a_{2,1}. \]

(16)

– Expression to compute the effective ink concentration

\[ a_{2,1}(ink_t) = a_{2,p}(ink_t) + \xi_{2,1}(ink_t), \]

(17)

Where \( ink_t \) : the theoretical concentration, one of \( c = [c_t, m_t, y_t, k_t] \)
Empirical function

- To represent the variations of the dot gain function due to the previous ink

\[
\xi_{2,1}(\text{ink}_t) = H_{2,1} \exp\left(-\frac{(\text{ink}_t - \mu_{2,1})^2}{s_{2,1}}\right) \text{ink}_t (1-\text{ink}_t) \tag{18}
\]

Where \( H, \mu, \) and \( s \) : parameters

Figure 3. Ink variation model [Eq. (15)] for different values of \( H(a), \mu(b) \) and \( s(c) \)
Optical dot gain

- Theoretically modeled by the Yule-Nielsen coefficient

- Values of $n$
  - Physical meaning for $n<2$
  - Approach
    - Any value that minimizes the accuracy error of the model
    - Treating as an optimization parameter, regardless of its physical meaning

- Performance simultaneously with n-value and dot gain function optimization
Parameter of the printer model
  - 159 parameters

The cost function to minimize

\[
\text{cost} = \frac{1}{S} \left[ \sum_{s=1}^{S} \frac{1}{\Gamma} \left( \sum_{\lambda=1}^{\Gamma} (R_{\text{print},\lambda,s} - R_{\text{meas},\lambda,s})^2 \right) \right]
\]

(19)

Where
- \( S \) : the number of samples in the training set
- \( \Gamma \) : the number of wavelength samples
- \( R_{\text{print}} \) : the reflectance computed with the YNSN model
- \( R_{\text{meas}} \) : the measured reflectance of the training set
Experiments

◆ Environment
  – Epson StylusColor™ 740 ink-jet printer
  – Epson StylusPhoto™ 890 ink-jet printer
    • Using from a six-ink printer to a four-ink printer
  – Epson Photo Quality paper
  – Floyd Steinberg dithering
  – Training set
    • Uniform ramps of eleven patches each
    • 143 samples
– Training set sheets

Figure 4. The set of images to obtain measured data for the training phase.

- Step1: Printing with the 125 patches of the training set
- Step2: The patches for overprints measurements on the right
- Step3: The print of the second image on the same sheet with black
- Step4: The overprint of the third with the three inks c, m, y
- Step5: The overprint of the last image with the four inks
– Test set
  • 777 samples in the HSV color space
  • Measurements of the spectra with a Gretag Spectrolino
  • the wavelength range from 400 to 700nm with a step of 10nm
– The “GAdeme” genetic algorithm in the Galib library
  • For the optimization phase
  • Computationally very expensive
  • Long time of the performance
Figure 5. A graphical representation of the model
### Table III. Error statistics for the proposed method.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>$\Delta E_{ab}^*$ Avg.</th>
<th>$\Delta E_{ab}^*$ 95%</th>
<th>rms% Avg.</th>
<th>rms% Max.</th>
<th>rms% S. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epson Stylus Color 740</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training</td>
<td>1.471</td>
<td>4.434</td>
<td>0.495</td>
<td>2.176</td>
<td>0.451</td>
</tr>
<tr>
<td>Test</td>
<td>1.541</td>
<td>3.956</td>
<td>0.585</td>
<td>2.397</td>
<td>0.461</td>
</tr>
<tr>
<td>Epson Stylus Photo 890</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training</td>
<td>1.407</td>
<td>4.443</td>
<td>0.499</td>
<td>2.354</td>
<td>0.459</td>
</tr>
<tr>
<td>Test</td>
<td>2.040</td>
<td>5.165</td>
<td>1.019</td>
<td>2.932</td>
<td>0.485</td>
</tr>
</tbody>
</table>

### Table IV. Error statistics when considering a single dot gain function for each ink.

<table>
<thead>
<tr>
<th>Data set</th>
<th>$\Delta E_{ab}^*$ Avg.</th>
<th>$\Delta E_{ab}^*$ 95%</th>
<th>rms% Avg.</th>
<th>rms% Max.</th>
<th>rms% S. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epson Stylus Color 740</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training</td>
<td>2.232</td>
<td>5.943</td>
<td>1.285</td>
<td>6.706</td>
<td>1.032</td>
</tr>
<tr>
<td>Test</td>
<td>2.629</td>
<td>5.500</td>
<td>1.675</td>
<td>5.952</td>
<td>1.033</td>
</tr>
<tr>
<td>Epson Stylus Photo 890</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training</td>
<td>2.057</td>
<td>5.587</td>
<td>0.908</td>
<td>3.579</td>
<td>0.830</td>
</tr>
<tr>
<td>Test</td>
<td>2.566</td>
<td>5.876</td>
<td>1.242</td>
<td>4.391</td>
<td>0.698</td>
</tr>
</tbody>
</table>
Figure 6. Dot gain functions for the two printers tested for the cyan (a), yellow (b), magenta (c), and black inks (d).
Epson Stylus Color 740
dot gain functions

Epson Stylus Photo 890
dot gain functions
Figure 7. The Yule-Nielsen n-value estimated in the optimization for the Epson StylusColor 740 printer and the Epson StylusPhoto 890 printer.
Conclusions

◆ Comparison between a single dot gain function and our approach
  – Our approach
    • Epson StylusColor740: Average rms: 0.59%, 1.54 $\Delta E_{ab}^*$
    • Epson StylusPhoto 890: Average rms: 1.02%, 2.04 $\Delta E_{ab}^*$
  – In case of single dot gain function
    • Epson StylusColor740: Average rms: 1.68%, 2.63 $\Delta E_{ab}^*$
    • Epson StylusPhoto 890: Average rms: 1.24%, 2.57 $\Delta E_{ab}^*$

◆ Future research
  – An extension of proposed model
◆ Rotated screens
  – Placing of the dots for each separation at different angles
  – $75^\circ$, $15^\circ$, $0^\circ$, and $45^\circ$ for cyan, magenta, yellow, and black
  – $0^\circ$: to the right
  – Advantage: reducing of the color error
    • Occur due to slight misregistration of the various separations
  – Disadvantage: suffer from large-area spatial defects
    • Such as moire’

◆ Dot-on-dot screens
  – Placing of dots for all separations in the same place
    • On a fixed grid
Fig. (a) and (c) are typical rotated screens; (b) and (d) show dot-on-dot halftoning. (a) and (b) are from the same, properly-registered page. (c) and (d) are from a page with severely misregistered separations. This misregistered yellow separation has imposed a color difference of about $12 \Delta E^*_{ab}$ between patches (b) and (d). patches (a) and (c) are only $107 \Delta E^*_{ab}$ different. The actual width of each image is approximately 1mm.
◆ Stochastic screens
  – To minimize the perceived difference between the binary image output and the original continuous tone image
  – Evaluating the perceived error on the basis of the human contrast sensitivity
  – Area coverage
    • Distributing over the entire cell