Computational Surface Models for Chromaticity Differences

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Abstract

**Computational surface model**
- Equal perceived chromaticity difference
- Based on the surfaces defined by an ellipse data set
- Calculated by a method
  - Weighted distance transform on curved space
- Using MacAdam ellipses
- Compared with CIE DE2000
Introduction

- Equal chromaticity
  - MacAdam chromaticity difference ellipse
  - Perceived equally in the ellipse

Fig. 1. The MacAdam ellipse. The Axes of plotted ellipses are 10 times Their actual lengths.
– Recently color difference formulas
  • Measured at various illumination levels
– Compensate for the lack of uniformity of the CIELAB color space
– Latest CIE recommended color difference formula
  • CIE DE2000
The purpose
- Computational surface model
- Focused only on the chromaticity differences measurements
- Derived from the chromaticity ellipses
- Three computational models
  - The mixing model
  - The line model
    - The line model with two pairs of planes
    - The line model with chromaticity difference grid
1. The mixing model

- Using all 25 ellipses from the MacAdam
- Projected from the center of projection above each circle
  - At the same height $H$ from the xy-plane
  - Making surface depending on the ellipses

Fig. 2. (a) projection of the circle, major semiaxis (b) minor semiaxis
– Two surfaces

Fig. 4. Defining surface $S$ from the surfaces $S_a$ and $S_b$

- Mixture of these two surfaces

\[ S = pS_b + (1-p)S_a \]  

where \[ p : p(\theta_1, \theta_2), \ 0 \leq p \leq 1 \]
– Surface $S_a$ and $S_b$

Fig. 5. Surface $S_a$ and $S_b$, which was elevated for visualizing
– Calculation of the parameter $p$

$\theta_1, \theta_2, \phi$

Fig. 3. Angles $\theta_1, \theta_2$ and $\phi$ in the calculation of the parameter $p$

• Parameter $p$

$$\text{if } \phi \leq \frac{\pi}{2} \quad p = \frac{\phi}{\frac{\pi}{2}}$$  \hspace{1cm} (3)

$$\text{elsewise } \quad p = \frac{|\phi - \pi|}{\frac{\pi}{2}}$$  \hspace{1cm} (4)
– Surface S

\[ (x_0, y_0) = (0.304, 0.433) \]
\[ (x_1, y_1) = (0.314, 0.453) \]

**Fig. 6.** Surface S according to the two chromaticities

– Chromaticity difference

\[ \Delta E = f(S) \quad \text{where} \quad f(S): \text{distance along the surface } S \quad (5) \]
2-1. The Line Model (LMPP)

- The line model with two pairs of planes
  - Parallel projections of circles
  - A two dimensional model
    - The longest semiaxis of all the ellipses
      - Horizontal

Fig. 7. The definition of the curve in the two dimensional model
– Three dimensional surface model
  • Three attribute of the ellipse
    – The a- and b- semiaxes and rotation angle $\theta$
  • Using a- and b-semiaxis

Fig. 8 (a). a-direction and b-direction on the xy-plane
– The plane is rotated vertically along a- and b-semiaxix.

$$\alpha_b = \cos^{-1}(a/r)$$  \hspace{1cm} (6-a)

$$\alpha_a = \cos^{-1}(b/r)$$  \hspace{1cm} (6-b)

$$d \cdot \tan(\alpha_a)$$  \hspace{1cm} (7)

Fig. 8 (b). The definition of heights in the three dimensional model.
- The plane

Fig. 9. A 3D illustration of the $S_a$ and $S_b$ surface

- Chromaticity difference

$$\Delta E = \sqrt{(d_a \cdot \cos(\varphi))^2 + (d_b \cdot \sin(\varphi))^2}$$  \hspace{1cm} (8)

- Ellipse parameterization

$$x = a \cdot \cos(\alpha), \quad y = b \cdot \sin(\alpha)$$  \hspace{1cm} (9)
2-2. The Line Model (LMCD)

- The line model with chromaticity difference grid
  - Chromaticity difference
    - Calculated from the starting point to all other points
  - Based on three parameters of an ellipse
    - The a- and b-semiaxes and rotation angle $\theta$

Fig. 8. The visualization of the ellipse parameters.
– Definition of the surface

\[
\alpha_i = \cos^{-1}\left(\frac{c_i}{r}\right)
\]

\[
\cos(\alpha_i) = \frac{c_i}{r}
\]

\[
C(i) = d_i \cdot \tan(\alpha_i)
\]

\[
d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}
\]

Fig. 10. (a) Height of the each point (b) chromaticity difference grid (c) surface
– Chromaticity difference

\[ c_i = \sqrt{(a_i \cdot \cos(\beta))^2 + (b_i \cdot \sin(\beta))^2} \]

where \( \tan \beta = \frac{a_0}{b_0} \cdot \tan(\varphi) \)

– Total chromaticity difference

\[ \Delta E = \frac{D_s}{r} \] (11)

where \( D_s \) : the shortest calculated distance
\( r \) : reference axis (the longest chromaticity difference)
Calculating the distance on a curve surface

- Weighted distance transform on curved space
  - The shortest distance between two points along the surface
  - 4-neighbor pixel

Fig. 12. The kernel for the WDTOCS calculation
First iteration

\[
F^*(e) = \min[F(e), \min[da + F^*(a), db + F^*(b),
\quad dc + F^*(c), dd + F^*(d)]]
\]

where

\[
da = \alpha \sqrt{(g(e) - g(a))^2 + \beta},
\quad db = \alpha \sqrt{(g(e) - g(b))^2 + \delta}
\]
\[
dc = \alpha \sqrt{(g(e) - g(c))^2 + \beta},
\quad dd = \alpha \sqrt{(g(e) - g(d))^2 + \delta}.
\]

\(G(x)\) : original digital surface

\(F(x)\) : binary image

\(\alpha = 1, \beta = 2, \delta = 1\)
– Second iteration

\[ F^*(e) = \min[F(e), \min[df + F^*(f), dg + F^*(g), dh + F^*(h), dk + F^*(k)]] \] (13)

where

\[
\begin{align*}
    df &= \alpha \sqrt{(g(e) - g(f))^2 + \delta}, \\
    dg &= \alpha \sqrt{(g(e) - g(g))^2 + \beta}, \\
    dh &= \alpha \sqrt{(g(e) - g(h))^2 + \delta}, \\
    dk &= \alpha \sqrt{(g(e) - g(k))^2 + \beta}.
\end{align*}
\]

- \( G(x) \): original digital surface
- \( F(x) \): binary image
- \( \alpha = 1, \beta = 2, \delta = 1 \)
Experiments

- **Ellipse data set**
  - Four different color discrimination data set

<table>
<thead>
<tr>
<th>TABLE I. Ellipse Data Sets</th>
<th>Dataset</th>
<th>No. of ellipses</th>
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<tbody>
<tr>
<td>total</td>
<td>82</td>
<td></td>
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<tr>
<td>BFD</td>
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<tr>
<td>MMB</td>
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<td>Rit-DuPont</td>
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<tr>
<td>Witt</td>
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<tr>
<td>CIE DE2000 dataset</td>
<td>total</td>
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- Calculated from the center of the ellipse to the edge of the ellipse
Fig. 13. Chromaticity difference when angle is $0^\circ$ and $90^\circ$

TABLE II. Summary of the Chromaticity Difference Calculations from the MacAdam Dataset

<table>
<thead>
<tr>
<th></th>
<th>MM</th>
<th>LMPP</th>
<th>LMCD</th>
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</thead>
<tbody>
<tr>
<td><strong>Angles $\varphi$ 0° and 90°</strong></td>
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<td></td>
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<tr>
<td>arithmetic mean</td>
<td>0.257</td>
<td>1.0054</td>
<td>0.9976</td>
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<td>standard deviation</td>
<td>0.180</td>
<td>0.0258</td>
<td>0.0070</td>
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<td><strong>Angles $\varphi$ 22.5°, 45° and 67.5°</strong></td>
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<tr>
<td>arithmetic mean</td>
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<td>1.0042</td>
<td>0.9991</td>
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<tr>
<td>standard deviation</td>
<td>0.139</td>
<td>0.0162</td>
<td>0.0098</td>
</tr>
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</table>
Experimental results from the CIE DE2000 data set

- Color difference
  - Calculated from the center of the ellipse to the edge of the ellipse

- Data set

Fig. 15. The numbers of the chosen ellipses from the CIE DE 2000 data set.
Fig. 16. Relative chromaticity differences between this work and CIE DE2000
Comparison with two measure

Table 3. Summary of the chromaticity difference calculations from the CIE DE 2000 dataset

<table>
<thead>
<tr>
<th></th>
<th>LMCD</th>
<th>CIE DE2000</th>
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<tr>
<td></td>
<td>Ellipse</td>
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<td>Standard deviation</td>
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<td>Small scale</td>
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<tr>
<td>all</td>
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Conclusions

- New computational surface models
  - Using MacAdam ellipse
  - Calculating chromaticity difference
  - Comparison with CIE DE2000 color difference
    - Other ellipse data set
  - Good performance
    - LMCD method