Dot pattern generation technique using molecular dynamics

T. Ide, H. Numata, and Y. Taira

School of Electrical Engineering and Computer Science
Kyungpook National Univ.
Abstract

◆ New technique for generating homogeneously distributed irregular dot patterns
  – Introducing irregularity by using low-discrepancy sequences
  – Improving the distribution of dots by using molecular dynamics
  – Illustrating the quality of dot patterns calculated with our approach
  – Applying to liquid-crystal displays
Introduction

◆ The edge-lit backlight unit
  – Conventional structure of edge-lit backlight unit

Fig. 1. There is a pattern of diffusing white spots on the bottom surface of the light guide. (a) The x and y axes, which are perpendicular to each other, show the directions of the prismatic groove of the prism sheets. (b) An example of the path of a light ray is shown.
Pseudorandom perturbation (PRP) approach
  – Based on pseudorandom numbers
  – Unsuitable for the recent high-performance backlight units and digital halftoning

Requirements for generating irregular dot patterns
  – Properly irregular so as not to cause any moire patterns
  – Sufficiently uniform not to cause visible roughness
  – Being capable of providing any density gradation
Low-discrepancy sequences

- Discrepancy
  - Definition of discrepancy

\[
D_{N}^{(2)} = \sup_{(x,y) \in [0,1]^2} \left| \frac{\#E(x, y)}{N} - xy \right|
\]  

- \(\#E(x, y)\) represents the number of points within a rectangular domain \(E(x, y) = [0, x] \times [0, y]\)
- \(N\) represents the total number of points
– Other definition of discrepancy for actual dot patterns

\[
T_{N}^{(2)} = \left\{ \int\int_{[0,1]^2} \left[ \frac{\#E(x, y)}{N} - xy \right]^2 dxdy \right\}^{\frac{1}{2}}
\] (2)

\[
T_{N}^{(2)} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ 1 - \max(x_i, x_j) \right] \left[ 1 - \max(y_i, y_j) \right] - \frac{1}{2N} \sum_{i=1}^{N} (1 - x_i^2)(1 - y_i^2) + \frac{1}{9}
\] (3)
Low-discrepancy sequences (LDS) and pseudorandom numbers

- LDS inequality

\[ D_N^{(2)} (\text{LDS}) \leq C \frac{(\log N)^2}{N} \]  \hspace{1cm} (4a)

- C is a \( N \)-independent constant

- The value of \( T_N^{(2)} \)

\[ T_N^{(2)} (\text{LDS}) = O\left( \frac{(\log N)^2}{N} \right) \]  \hspace{1cm} (4b)
– Results of pseudorandom numbers

\[ D_{N}^{(2)}(\text{random}) = O \left( \left( \frac{\log \log N}{N} \right)^{1/2} \right) \]  \hspace{1cm} (5a)

\[ \left\langle \left[ T_{N}^{(2)}(\text{random}) \right]^2 \right\rangle = \frac{\sqrt{5}}{6\sqrt{N}} \]  \hspace{1cm} (5b)

• \(<::\) denotes the expectation value
– Comparison between the LDS and pseudorandom numbers

Fig. 3. Comparison between (a) Pseudorandom numbers and (b) LDS.
Equation of motion

Equation of motion

\[ m \frac{d^2 r_i}{dt^2} + c \frac{dr_i}{dt} = \sum_{j=1}^{N} f_{ij}(r_i, r_j) \quad \text{for } i = 1, 2, ..., N \quad (6) \]

- \( m \) and \( c \) are constants
- \( t \) is a parameter of time
- \( r_i \) represents the coordinates \((x_i, y_i)\)
- \( f_{ij} \) means the repulsive force
Solution of the equation of motion

\[ r_i(t) = r_i(t_0) + \frac{1}{c} \int_{t_0}^{t} d\tau F_i(\tau) \left\{ 1 - \exp\left[ -\frac{c(t - \tau)}{m} \right] \right\} \] (7)

- \( F_{ij} \) is a shorthand notion for \( \sum_{j=1}^{N} f_{ij} \)
- \( t_0 \) is the initial time

Approximation of the solution

\[ r_i(t + \Delta t) = r_i(t) + \frac{1}{c} \Delta t F_i(t) \] (8)
Fig. 4. The repulsive force from dots B and C is acting on A. The situation is not one-sided: B and C are also affected by the surrounding dots, showing the permutation symmetry in the relaxation algorithm.
- The elliptic model

\[ f_{ij} = \frac{r_{ij}}{r_{ij}} \times \begin{cases} 1 & \text{for } b_{ij} < D \\ \exp\left[-\left|\frac{r_{ij} - b_{ij}}{L}\right]\right] & \text{for } b_{ij} \geq D \end{cases} \quad (9) \]

\[ \frac{b_{ij}^2}{(kD)^2} = \frac{x^2 + y^2}{(kx)^2 + y^2} \quad (10) \]

- L means the square length
Density distribution

- Introducing a new function

\[ P_k = \frac{\rho_k}{\sum_{l=1}^{M} \rho_l} \]  \hspace{1cm} (11)

- M denote the number of rectangles
- \( \rho_k \) denotes the density of each tile
Procedure for generating the initial pattern

- Generate a three-dimensional LDS defined within $[0,1]^3$, and take a point $(U_0, U_1, U_2)$
- Choose $k$ by using the condition

$$\sum_{l=1}^{k} P_l \leq U_0 < \sum_{l=1}^{k+1} P_l$$

(12)

- Give the coordinates for the chosen tile by the equations

$$x = x_k + L_{xk} U_1, \quad y = y_k + L_{yk} U_2$$

(13)

- $x_k$ and $y_k$ are the origin coordinates of the $k$th tile
– Principal length

\[ \lambda(\rho) = \frac{a}{\sqrt{\rho}} \]  \hspace{1cm} (14)

• a is diameter of dots

– Imposing a scaling rule

\[ D \sim \lambda(\rho) \]  \hspace{1cm} (15)
Calculated results

- Dynamical LDS (DLDS)
  - Comparison between two initial patterns

Fig. 5. (a) Pseudorandom numbers and (b) LDS. The iteration number and the dynamical parameters are the same for both. We observe visible roughness in (a) and uniform irregularity in (b).
Example of DLDS patterns with steep density gradient

Fig. 6. (a) DLDS pattern and (b) Density distribution.
– Squared discrepancy as a function of N

Fig. 7. Square of the $L_2$ discrepancy as a function of N.
Application to LCD

- PRP method and DLDS method
  - Prototyped integrated-type light guides

Fig. 8. (a) Prototyped integrated-type light guides. Shape of (b) The microscatterers and (c) The prismatic grooves in units of millimeters.
– Snapshots of the microscatterer patterns

Fig. 9. Use of (a) The PRP method and (b) The DLDS method.
– Snapshots through a liquid-crystal cell

Fig. 10. Use of (a) The PRP method and (b) The DLDS method.
– Luminance fluctuation along the vertical axis in Fig.11 for light guides

Fig. 11. Use of (a) The PRP method and (b) The DLDS method. For (a) and (b), the luminance is normalized with the luminance value at the base of Figs. 11(a) and 11(b), respectively.
Conclusions

- New approach to generate irregular dot patterns
  - Showing the important role of a LDS in generating the physical dot patterns
  - Developing an effective algorithm to remove interdot overlap and to provide continuous-density variation
  - Generating superuniform irregular dot patterns even under the condition of a steep density gradient
Homogeneity and irregularity

- Applied to the back-light units of LCDs
- Eliminating a moire pattern
- Improving the luminance homogeneity