Estimation of Illumination Characteristics

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Abstract

- Estimating the parameters which describing the evolution of the changing illumination condition
  - Introducing the usage of conical coordinate system in color space
  - Deriving the differential equation describing the illumination change
  - Illustrating the algorithm with some simulation
1. Introduction

- Color constancy
  - Being given the image $I^{(0)}$ of a scene $R$ under $I^{(0)}$ illumination
    - $I^{(0)} = I(R, l^{(0)})$ and another illumination $l^{(1)}$ estimate $I^{(1)} = I(R, l^{(1)})$
    - Only the image data $I^{(0)}$ and the illumination $l^{(1)}$ is given
– Assuming that we can observe under a continuously changing illumination $I(t) = I(R_l(t))$
  - Describing the change of the color vector by partial differential equation with polynomial coefficient

– Notations

- $E$ : identity matrix
- $\lambda$ : wavelength variable
- $M$ : matrices
- $V$ : vector space
- $s$ : vectors

– Assuming that we know the reflectance spectra of the scene points
2. Conical structure of spectral color space

- Conical structure
  - Linear combinations of a few basis vectors
    \[ s(\lambda) \approx \sum_{k=0}^{K} \sigma_k b^k(\lambda) \]  
    
    \( s(\lambda) \): spectral vector  
    
    \( b^k \): basis vector  
    
    \( \sigma_k \): coefficient
  - These vectors \( \sigma \) are located in a cone
    \[ c = \{ (\sigma_0, \sigma_1, \sigma_2) : \sigma_0^2 - \sigma_1^2 - \sigma_2^2 \geq 0 \} \]
Fig. 1. (a) First three eigenvectors from spectral database
(b) Distribution of the coefficient vectors
- A measurement of the grayness of the spectrum

\[
\| \sigma \| = \sigma_0^2 - \sigma_1^2 - \sigma_2^2 \cdots - \sigma_N^2 \geq 0
\] (2)

- The linear transformations which preserve the grayness form the group \( SU(1,1) \)

- Forming \( SU(1,1) \) by the complex \( 2 \times 2 \) matrices of the form

\[
M = \left( \begin{array}{cc} a & b \\ \bar{b} & \bar{a} \end{array} \right) \text{ with } |a|^2 - |b|^2 = 1
\] (3)

- Defining the group \( SU(1,1)^+ = R \times SU(1,1) \)

\[
SU(1,1)^+ = \{(e^\sigma, M) : \sigma \in R, M \in SU(1,1)\}
\]

- Group operation

\[
(\sigma_1, M) \circ (\sigma_2, M) = (\sigma_1 + \sigma_2, M_1 M_2)
\] (4)
3. One parameter groups and differential operators

- Differential operators in Lie group
  - Imitating traditional color science notation

\[
L = \sigma_0, \quad A = \frac{\sigma_1}{\sigma_0}, \quad B = \frac{\sigma_2}{\sigma_0} \quad (5)
\]

\[
\tau = \log L \quad \text{and} \quad z = A + iB \quad (6)
\]

- Partial differential operators \( D_k \) in the \( \sigma \) coordinate

\[
D_k f(\sigma_0, \sigma_1, \sigma_2) = \frac{\partial f(\sigma_0, \sigma_1, \sigma_2)}{\partial \sigma_k}, \quad k = 0, 1, 2 \quad (7)
\]
- The group $SU(1,1)^+$ acts on LAB-space

$$(\sigma, M)(L, A, B) = (\sigma, M)(L, z)$$

$$= (e^{\sigma L}, M z + b)$$

(8)

- Connection between subgroups and differential operators

  - Taking the subgroups $g_0 = \{ (\sigma, E): \sigma \in \mathbb{R} \}$ defining the mapping $(L, z) \rightarrow (\sigma, E)(L, z) = (e^{\sigma L}, z)$

  $$f \rightarrow D_{g_0} f = \frac{\partial}{\partial \rho} f((\rho, E)(L, z)) \bigg|_{\rho = 0}$$

  $$= \frac{\partial}{\partial \rho} f (e^\rho L, z) \bigg|_{\rho = 0}$$

  (9)

  $$= \frac{\partial}{\partial \rho} f (e^\rho \sigma_0, e^\rho \sigma_1, e^\rho \sigma_2) \bigg|_{\rho = 0}$$

  $$= \sigma_0 D_0 f + \sigma_1 D_1 f + \sigma_2 D_2 f$$
- **Theorem 1**

  - The difference operator $D_g$ defined through the one parameter subgroups $g$ of $SU(1,1)^+$ form a vector space of dimension four.

  - Four basis vector

    \[ g_0 = R \]
    \[ g_1 = \begin{pmatrix} 0, & \begin{pmatrix} \cosh(\alpha / 2) & \sinh(\alpha / 2) \end{pmatrix} \\ \sinh(\alpha / 2) & \cosh(\alpha / 2) \end{pmatrix} \right\} = \{(0, M_1(\alpha))\} \]
    \[ g_2 = \begin{pmatrix} 0, & \begin{pmatrix} \cosh(\alpha / 2) & i \sinh(\alpha / 2) \end{pmatrix} \\ -i \sinh(\alpha / 2) & \cosh(\alpha / 2) \end{pmatrix} \right\} = \{(0, M_2(\alpha))\} \]
    \[ g_3 = \begin{pmatrix} 0, & \begin{pmatrix} e^{i\varphi / 2} & 0 \\ 0 & e^{-i\varphi / 2} \end{pmatrix} \right\} = \{(0, M_3(\alpha))\} \]
- Differential operators

\[ D_{g_0} = \sigma_0 D_0 + \sigma_1 D_1 + \sigma_2 D_2 \]
\[ D_{g_1} = \frac{\sigma_0^2 - \sigma_1^2 + \sigma_2^2}{2\sigma_0} D_1 - \frac{\sigma_1 \sigma_2}{\sigma_0} D_2 \]
\[ D_{g_2} = -\frac{\sigma_1 \sigma_2}{\sigma_0} D_1 + \frac{\sigma_0^2 - \sigma_1^2 + \sigma_2^2}{2\sigma_0} D_2 \]
\[ D_{g_3} = \sigma_1 D_2 - \sigma_2 D_1 \]

- One-parameter subgroup \( g \) of \( SU(1,1)^+ \) defines a curve in \( (\sigma_0, \sigma_1, \sigma_2) \) space and by differentiation an operator \( D_g \)

\[ D_g = a_0 D_{g_0} + a_1 D_{g_1} + a_2 D_{g_2} + a_3 D_{g_3} \]
- Assuming that the function $f(\sigma_0(t), \sigma_1(t), \sigma_2(t))$ describes our measurements varying over some period of time

$$D_g f = \left. \frac{\partial f(\sigma_0(t), \sigma_1(t), \sigma_2(t))}{\partial t} \right|_{t=0}$$

(13)

$$\left. \frac{\partial f(\sigma_0(t), \sigma_1(t), \sigma_2(t))}{\partial t} \right|_{t=0} = D_g f$$

(14)

$$= a_0 D_{g_0} f + a_1 D_{g_1} f + a_2 D_{g_2} f + a_3 D_{g_3} f$$

- Computing $D_g f, D_{g_k} f, (k=0..3)$ the measured data and being able to estimating the unknown constants $a_0, ..a_3$
- Theorem 2

- A basis of the Lie-algebra $\text{su}(1,1)^+$ in LAB coordinates is given by the operators

\[
D_{g_0} = LD_L
\]
\[
D_{g_1} = \frac{(1 - A^2 + B^2)D_A - (AB)D_B}{2}
\]
\[
D_{g_2} = \frac{(1 + A^2 - B^2)D_B - (AB)D_A}{2}
\]
\[
D_{g_3} = -BD_A + AD_B
\]

where $D_L, D_A, D_B$ are the differential operators

\[
D_L f = \frac{\partial f(L, A, B)}{\partial L}, \quad D_A f = \frac{\partial f(L, A, B)}{\partial A}, \quad D_B f = \frac{\partial f(L, A, B)}{\partial B}
\]
4. Application to illumination invariant recognition

- Showing how the general theory can be used to recover characteristic illumination parameter
  - Simple model of the image processing

\[ m_k(x) = \int l(\lambda) r(x, \lambda) c^{(k)}(\lambda) d\lambda \]  

(16)

where

- \( m_k(x) \): value measured with sensor number \( k \) at position \( x \)
- \( l(\lambda) \): illumination spectrum
- \( r(x, \lambda) \): reflectance spectrum at position \( x \)
- \( c(\lambda) \): spectral characteristics of an imaging sensor
\[ r(x, \lambda) \approx \sum_{v=0}^{2} b^{(R)}_v(\lambda) \sigma^{(r)}_v x \]

\[ l(\lambda) \approx \sum_{u=0}^{2} b^{(I)}_u(\lambda) \sigma^{(l)}_u \]

where \( b^{(R)}_v(\lambda), b^{(I)}_u(\lambda), v = 0,1,2 \) is the basis vector
\( \sigma^{(r)}_v, \sigma^{(l)}_u \) is the coordinate vectors

\[ m_k(x) \approx \sum_{v=0}^{2} \sum_{u=0}^{2} \sigma^{(r)}_v(x) \sigma^{(l)}_u \int b^{(r)}_v b^{(l)}_u c^{(k)}(\lambda) d\lambda \]

\[ = \sum_{v=0}^{2} \sum_{u=0}^{2} \sigma^{(r)}_v(x) \sigma^{(l)}_u \gamma^{(k)}_{uv} = \sigma^{(r)}(x)' G^{(k)} \sigma^{(l)} \]

where \( G^{(k)} \) characterizes channel \( k \) of the sensor
\[ m_{k,x}(\sigma) = v_k(x)' \sigma = \langle v_k(x), \sigma \rangle \] (18)

\[ f(\sigma_0, \sigma_1, \sigma_2) = f_{k,x}(\sigma_0, \sigma_1, \sigma_2) = \langle v_k(x), \sigma \rangle = m_{kx}(\sigma) \] (19)

- Observing the same point with the same camera under changing illumination conditions produces a measurement series \( m_{k,x}(\sigma, t) \)
- Computing the time derivative

\[ m_{k,x}(\sigma, t) = \frac{\partial m_{k,x}(\sigma, t)}{\partial t} \] (20)

- The mapping \( m \rightarrow m' \) defines a differential operator \( D_g \)
  which is a linear combination of the known differential operators \( D_{g_k} \) with unknown coefficients \( a_k \)
- The algorithm to recover the unknown illumination parameters
  
  - Select a number of points in the image
  - For each point sensor combination $\pi = (x, k)$ measure the sequence $f_{k,x}(\sigma_0(t), \sigma_1(t), \sigma_2(t)) = f_\pi(t)$
  - For each point/sensor combination $\pi$ compute the derivative
    \[ m'_\pi = D_{g_f} f_\pi = \left. \frac{\partial f_{k,x}(\sigma_0(t), \sigma_1(t), \sigma_2(t))}{\partial t} \right|_{t=0} \]
    
    - Collect all values in the vector $m = (m_\pi)$
    - For each point/sensor combination $\pi$ compute the derivatives
      \[ D_{g_k} f_\pi = u_{k\pi} \]
For each point/sensor combination \( \pi \) compute the derivatives and collect them in the matrix \( U \)

\[
D_{g_k} f_{\pi} = u_{k\pi} \quad k = 0, 1, 2, 3
\]

Between each row \( u_{\pi} \) of \( U \) and the corresponding element \( m_{\pi} \) in \( m \)

\[
m_{\pi} = u_{\pi} a
\]

where the vector \( a \) collect the unknown coefficients \( a_0 \ldots a_3 \)

The unknown coefficient vector can be estimated by solving the equation

\[
m = Ua
\]  
\[(21)\]
– Use of the LAB coordinate system

- Tracking the LAB values at a given position over a period of time defines functions $L(t, x), A(t, x), B(t, x)$
- Defining $L'(x), A'(x), B'(x)$ as the time derivatives of $L, A$ and $B$ at $t = 0$

$$L' = a_0 L$$
$$A' = \frac{(1 - A^2 + B^2)}{2} a_1 - \frac{a_2}{2} (AB) - a_3 B$$
$$B' = -\frac{(1 + A^2 - B^2)}{2} a_2 + \frac{a_1}{2} (-AB) + a_3 A$$  \hspace{1cm} (22)$$
5. Experiments

- Testing the algorithm by simulating a sequence of images
  - Using multispectral image in these experiments
  - Using the spectral characteristics of a CCD camera for a given multispectral image
  - Deriving the basis vectors in the space of illumination spectral by generating a random mixture of 1000 spectra consisting the daylight spectral and three artificial daylight spectra
Generating a time varying series of illumination spectra

- Illumination light is characterized by the flat illumination at $t = 0$
- Calculating the camera image from the original multispectral image and camera characteristics
- $t = 1$, the illumination is given by a pre-defined spectrum
- Calculating the camera images showing the scene at equidistant points in time
- Calculating discrete versions of the time-derivative at points in image
- Using three equations

\[
L' = a_0 L
\]

\[
A' = \frac{(1 - A^2 + B^2)}{2} a_1 - \frac{a_2}{2} (AB) - a_3 B
\]

\[
B' = -\frac{(1 + A^2 - B^2)}{2} a_2 + \frac{a_1}{2} (-AB) + a_3 A
\]
– Two decision problem
  - Deciding which points should be tracked
    - Random collection
    - The points with the highest intensity in the multispectral images
  - Deciding how each of the points tracked contribute to the final estimation of the parameter values
    - The mean of the individual estimates
    - A weight for the estimated vector by using determinant of the matrix $v(x)$
- First experiment
  - Using the CIE-A source as the light source at time $t = 1$ and multispectral image “inlab2”
  - Three frames are collected at $t = 0.2, 0.3, 0.4$
  - Tracking the ten points with highest intensity
  - Computing mean- and the weighted-sum of the estimated parameter vectors

- Second experiment
  - Using the same parameter except the D65 illuminant source
– Simulated camera image

Fig. 2. (a) “inlab2” camera image
(b) “rwood” camera image
(c) “ashton2” camera image
Fig. 3. (a) The “inlab2” A-source, ten maximum intensity points tracked, three frames at t=0.2,0.3,0.4
(b) The “inlab2” D65-source, ten maximum intensity points tracked, three frames at t=0.2,0.3,0.4
Fig. 4. (a) The “inlab2” D65-source, ten random points tracked, three frames at $t=0.2,0.3,0.4$
(b) The “ashton” D65-source, ten maximum intensity points tracked, three frames at $t=0.2,0.3,0.4$
(c) The “rwood” D65-source, ten maximum intensity points tracked, three frames at $t=0.2,0.3,0.4$
6. Conclusion

- Deriving a set of partial differential equations which link the sequence of color images
  - Assuming that the object reflectance properties are known
  - No effect of the noise