Hybrid LMS-MMSE inverse halftoning technique

Pao-Chi Chang, Che-Sheng Yu, and Tien-Hsu Lee

School of Electrical Eng. & Computer Sci.
Kyungpook Nat’l Univ.
Abstract

- The objective of this paper
  - Reconstruction of high quality gray-level image

- Optimal inverse halftoning method
  - LMS adaptive filtering algorithm
  - Lookup tables designed by the MMSE method
    - Reduce the computational complexity
  - Hybrid LMS-MMSE inverse halftoning algorithm
1. Introduction

- Reconstruction process
  - Sliding window filtering process
  - Major parameter
    - Filter shape
    - Filter coefficients
    - Filter order
  - LMS algorithm
    - Low complexity
    - Excellent performance
- MMSE table lookup method
  - Good reconstructed quality and fast speed
  - Empty cell problem

- Hybrid LMS-MMSE method
  - MMSE lookup table method is first choice
  - LMS method is used in empty cell
  - Hybrid method yields the best performance
2. Inverse halftoning by LMS adaptive filtering

- Adaptive algorithm
  - The structure of the training process

Fig. 1. Block diagram of the LMS adaptive filtering algorithm
– The reconstructed image

\[ \hat{g}(i, j) = \sum_{(k,l) \in M} w(k,l) b(i-k, j-l) \]

where

\[ w(k,l) \quad \rightarrow \quad \text{Filter weight} \]
\[ M \quad \rightarrow \quad \text{Filter mask} \]
\[ b(i-k, j-l) \quad \rightarrow \quad \text{Bi-level halftoned pixel} \]

– The reconstruction error

\[ e(i, j) = g(i, j) - \hat{g}(i, j) \]
- Weight adaptation

\[ w_{m+1}(k, l) = w_m(k, l) + 2\mu e(i - k, j - l)b(i - k, j - l) \]

\[ \rightarrow \quad \text{all}(k, l) \in M \]

where

\[ m \rightarrow \text{Iteration index} \]

\[ \mu \rightarrow \text{Parameter of updating step size} \]
Optimal mask shapes

- The use of $7 \times 7$ square mask
- Training the reconstruction filter
- Eliminate the least significant weight
- Observe the mask shape
(a) Clustered-dot dither  
(b) Dispersed-dot dither  
(c) Error diffusion

Fig. 2. Variation of the LMS mask shape and PSNR values
Fig. 3. Coefficients of the filters designed by the LMS method

(a) Clustered-dot dither

(b) Dispersed-dot dither

(c) Error diffusion
3. MMSE table lookup inverse halftoning

Algorithm

- The encoder and decoder
  - Mapping a gray level image to a binary halftoned image
  - Mapping an N-dimensional binary pattern to a gray level image
  - Optimal decoder
    - Centroid theorem

Mapping binary block to a gray level pixel
Centroid theorem

- Unique optimal codebook

\[ Y_i = E[X \mid X \in R_i] \]

Where

- \( R_i \rightarrow \) Set of original gray level value pixels that are mapped to \( i \)th binary pattern
- \( X \rightarrow \) Original gray level pixels
- \( Y_i \rightarrow \) Reconstructed gray level pixel value with respect to the halftone pattern \( i \)
– Construction of the lookup table

- Encoding mapping
  - Encode the gray level images by a given halftoned method
  - Tracking of the gray level value of the central point in a mask
  - Record the histogram $T_i(a)$
– Centroid calculation
  • Calculates the centroid $Y_i$ of the central point of a mask

  \[
  Y_i = \frac{\sum_{a=0}^{255} a \times T_i(a)}{\sum_{a=0}^{255} T_i(a)} \quad i = 0, 1, \ldots, 2^N - 1
  \]

– Table setup
  • Fill in the centroids into a table with $2^N$ entries
  • Design separately for different halftoned method
Experiments

- Evaluation of reconstruction quality
  - Mask size and mask shapes

(a) 9-point mask

Fig. 4. Halftone patterns and the corresponding histograms of the gray level distribution for the central pixel in clustered-dot dither
(b) 13-point mask

(c) 13-point mask
Table 1. PSNR (dB) of the reconstructed image with various mask size and halftone methods

<table>
<thead>
<tr>
<th>Image</th>
<th>Method</th>
<th>Mask Size</th>
<th>9</th>
<th>13</th>
<th>16</th>
<th>21</th>
<th>25</th>
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<td>25.39</td>
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</table>

**Table 1**: PSNR (dB) of the reconstructed image with various mask size and halftone methods.
- Results of the performance
  - MMSE method acquired best reconstruction quality
    ✓ Resultant optimal masks of the LMS method
    ✓ Optimal mapping
  - Empty cell problem
    ✓ No entry mapped to a specific halftone pattern
Table 2. Number of nonempty cells and empty cell fetches (in parentheses) of the MMSE method with various table sizes and halftone methods

<table>
<thead>
<tr>
<th>Image</th>
<th>Method</th>
<th>Table Size</th>
<th>$2^9=512$</th>
<th>$2^{13}=8192$</th>
<th>$2^{16}=65536$</th>
<th>$2^{21}=2097152$</th>
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<td>Clustered-dot</td>
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<td>500(0)</td>
<td>3307(118)</td>
<td>9064(358)</td>
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<td>286(22)</td>
<td>1494(119)</td>
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</table>
4. Hybrid LMS-MMSE inverse halftoning

Algorithm

Fig. 5. Flowchart of the hybrid LMS-MMSE inverse halftoning method
Procedure

- Determine the maximum order of the LMS adaptive filter
- Determine the maximum order of the MMSE method
- Determine the optimal mask shapes
- Build up the reconstruction table
- Determine the empty cell threshold
- Replace the empty cell table entry by the output of the M-point LMS filter
Experiments

(a) Gaussian filter (19.26dB)  (b) 21-point hybrid LMS-MMSE method (26.96dB)

Fig.6. Inversed halftoned images from clustered dot dither method
Fig. 7. Inversed halftoned images from dispersed dot dither method

(a) Gaussian filter (28.30dB)  (b) 21-point hybrid LMS-MMSE method (28.20dB)
Fig. 8. Inversed halftoned images from error diffusion

(a) Gaussian filter (30.41dB)  
(b) 21-point hybrid LMS-MMSE method (31.64dB)
Comparison of the reconstruction quality

- Hybrid LMS-MMSE approach is best perform
  • Error diffusion method

Table 3. Comparison of various inverse halftoning methods for the error diffusion kernel
5. Conclusion

- Hybrid LMS-MMSE method
  - Obtains optimal filter mask
  - MMSE table lookup method
    - Improves the reconstruction performance
    - Reduces the computational complexity
  - Solves the empty cell problem