Color Constancy

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Computation of lightness and color

- Computation of image intensity

\[ c(x, y) = R(x, y)L(x, y) \]  

(7.27)

where \( c(x, y) \) is image intensity,
\( R(x, y) \) is stand for reflectance at point \((x, y)\),
\( L(x, y) \) is stand for irradiance at point \((x, y)\).

- Taking logarithm of sensor’s response
  - Stage 1:

\[ o_1(x, y) = \log c(x, y) = \log R(x, y) + \log L(x, y) \]  

(7.28)

where \( o_1(x, y) \) is algorithm output generate by stage \( i \) with \( i \in \{1, \ldots, 5\} \).
• Stage 2
  
  − Edges in images
    » Sharp edges
    » Color changes
    » Smooth edges
      » Irradiance changes
  
  − Second stage using Laplacian symmetric filter

\[
o_2(x, y) = \nabla^2 (\log c(x, y))
    = \log R(x, y) + \log L(x, y))
    = \nabla^2 \log R(x, y) + \nabla^2 \log L(x, y)
\]

where \( \nabla^2 \log R(x, y) \) is nonzero only at positions where reflectance changes.

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\]

(7.29)

(7.30)
• Stage 3
  - Threshold operator

\[
\Theta(x) = \begin{cases} 
  x & \text{if } |x| > \theta \\
  0 & \text{if } |x| \leq \theta 
\end{cases}
\]  

(7.31)

where \( \theta \) determines the threshold.

- Applying threshold operator to output of Laplacian

\[
o_3(x, y) = \Theta(\nabla^2 \log R(x, y) + \nabla^2 \log L(x, y)) = \nabla^2 \log R(x, y)
\]  

(7.32)

\[
o_3(x, y) = \nabla^2 \log R(x, y) = \nabla^2 R'(x, y)
\]  

(7.33)

where \( o_3(x, y) \) is obtained by a suitably defined threshold, and, \( R' = \log R \), called poisson’s equation.
• Stage 4
  - Solving $R'$ by Green's function

  \[
  R'(x, y) = \iiint o_3(\xi, \eta)g(x - \xi, y - \eta)d\xi d\eta
  \]  

  » Substituting $o_3(\xi, \eta)$ by \( \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) R'(\xi, \eta) \)

  \[
  R'(x, y) = \iiint \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) R'(\xi, \eta)g(x - \xi, y - \eta)d\xi d\eta
  \]  

  \[
  = \iiint R'(\xi, \eta) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) g(x - \xi, y - \eta)d\xi d\eta
  \]  

  » Choosing $g(r) = \frac{1}{4\pi} \log(r^2)$ with $r^2 = x^2 + y^2$

  \[
  R'(x, y) = \iiint R'(\xi, \eta) \frac{1}{4\pi} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log((x - \xi)^2, (y - \eta)^2)d\xi d\eta
  \]
» And \( \frac{1}{4\pi} \nabla^2 \log(x^2 + y^2) = \delta(x, y) \), giving as:

\[
R'(x, y) = \iint R'(\xi, \eta) \delta(x - \xi, y - \eta) d\xi d\eta = R'(x, y)
\] (7.38)

» Selecting Kernel \( \frac{1}{4\pi} \log(r^2) \) to convolve output of Laplacian

\[
o_4(x, y) = \frac{1}{4\pi} \iint \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) R'(\xi, \eta) \log((x - \xi)^2 + (y - \eta)^2) d\xi d\eta
\] (7.39)

• Stage 5
  - Estimating of reflectances

\[
o_5(x, y) = \exp(o_4(x, y))
\] (7.40)
• Flowchart of algorithm

- Signal measured by sensors
- Edge detection using Laplacian
- Thresholding operation
- Integrate twice
- Exponentiate result
- Normalization
• Progressing of algorithm

Fig. 7.8 (a) Logarithm of input brightness, (b) edge detection using a Laplacian, (c) thresholding operation, and (d) integration. The impulses shown in (b) and (c) are caused by a change in reflectance. The thresholding operation removes small values that are caused by a change in the lighting conditions. The one-dimensional case is shown on the left.
- Processing on discretized images
  - Discretized Laplace operator

Fig. 7.9 (a) Two discretized versions of the Laplace operator. Care must be taken at the border of the image. The operators shown in the first column are the standard form of the Laplace operator. The second column shows the Laplace operator to be applied at the top/left corner of the image. The two rightmost operators are used at the top border of the image.
• Threshold processing
  - Threshold value with discretized Laplace filter (top of Fig. 7.9)
    » Horizontal and vertical filtering resulting
      \[ |o_2(x, y)| \leq 6\Delta \]
    » Diagonal filtering resulting
      \[ |o_2(x, y)| \leq 9\Delta \]
  » Threshold selecting
    » Separating change due to illuminant and reflectance
      \[ |o_2(x, y)| \leq 10\Delta \] (7.41)
- Threshold value with discretized Laplace filter (bottom of Fig. 7.9)
  » Horizontal and vertical filtering resulting
  \[
  |o_2(x, y)| \leq 1\Delta
  \]
  » Diagonal filtering resulting
  \[
  |o_2(x, y)| \leq 2\Delta
  \]
  » Threshold selecting
    » Separating change due to illuminant and reflectance
  \[
  |o_2(x, y)| \leq 3\Delta
  \] (7.42)

- Getting suitable threshold with logarithm of input values
  » Using filter top of Fig. 7.9
  \[
  |o_2(x, y)| \leq 9\log \left(1 + \frac{\Delta}{c}\right)
  \] (7.43)
  » Using filter bottom of Fig. 7.9
  \[
  |o_2(x, y)| \leq 3\log \left(1 + \frac{\Delta}{c}\right)
  \] (7.44)
Fig. 7.10 (a) Linear horizontal change in image brightness, (b) linear diagonal change in image brightness, (c) horizontal step due to a small change in image brightness, (d) diagonal step due to a small change in image brightness. The smallest change in image brightness resolved by the sensor is $\Delta$.
Undoing Laplacian

- Convolving output of Laplacian with \( \frac{1}{4\pi} \log r^2 \)

\[
o_3(x, y) = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) R'(x, y)
\]

(Solving for \( R'(x, y) \))

\[
o_3(x, y) = -20R'(x, y) + 4(R'(x + 1, y) + R'(x, y + 1) + R'(x - 1, y)
+ R'(x, y - 1)) + (R'(x + 1, y + 1) + R'(x - 1, y + 1) + R'(x - 1, y - 1) + R'(x + 1, y - 1))
\]

(Solving for \( R'(x, y) \))

\[
R'(x, y) = \frac{1}{5} (R'(x + 1, y) + R'(x, y + 1) + R'(x - 1) + R'(x, y - 1))
+ \frac{1}{20} (R'(x + 1, y + 1) + R'(x - 1, y + 1) + R'(x - 1, y - 1) + R'(x + 1, y - 1)) - \frac{1}{20} o_3(x, y)
\]
» Obtaining value of stage 4 by iterative processing

\[
o_{4}^{n+1}(x, y) = \frac{1}{5} (o_{4}^{n}(x + 1, y) + o_{4}^{n}(x, y + 1) + o_{4}^{n}(x - 1, y) + o_{4}^{n}(x, y - 1))
+ \frac{1}{20} (o_{4}^{n}(x + 1, y + 1) + o_{4}^{n}(x - 1, y + 1) + o_{4}^{n}(x - 1, y + 1) + o_{4}^{n}(x - 1, y - 1)) - \frac{1}{20} o_{3}(x, y)
\]  

(7.48)

- For smaller version of Laplacian(bottom of Fig. 7.9)

\[
o_{4}^{n+1}(x, y) = \frac{1}{4} (o_{4}^{n}(x + 1, y) + o_{4}^{n}(x, y + 1) + o_{4}^{n}(x - 1, y) + o_{4}^{n}(x, y - 1)) - \frac{1}{4} o_{3}(x, y)
\]  

(7.49)

» Condition of iterative finish

\[
\forall x, y \quad |o_{4}^{n+1}(x, y) - o_{4}^{n+1}(x, y)| < \varepsilon
\]  

(7.50)
Parallel implementation of Hron’s algorithm for color constancy

Fig. 7.11 Parallel implementation of Horn’s algorithm for color constancy. The individual steps are edge enhancement using a Laplacian and a threshold operation, followed by integration.
Using Jacobi’s method written in Gauss-Seidel formulation

\[
o_{4}^{n+1}(x, y) = \frac{1}{4} (o_{4}^{n}(x + 1, y) + o_{4}^{n}(x, y + 1) + o_{4}^{n+1}(x - 1, y) + o_{4}^{n+1}(x, y - 1) - o_{3}(x, y)) \tag{7.51}
\]

Corrective form

\[
o_{4}^{n+1}(x, y) = o_{4}^{n}(x, y) + \frac{1}{4} (o_{4}^{n}(x + 1, y) + o_{4}^{n}(x, y + 1)
+ o_{4}^{n+1}(x - 1, y) + o_{4}^{n+1}(x, y - 1) - o_{3}(x, y)) - o_{4}^{n}(x, y) \tag{7.52}
\]

\[
= o_{4}^{n}(x, y) + \text{Corrective – Term}(x, y) \tag{7.53}
\]

with:

\[
\text{Corrective – Term}(x, y) = \frac{1}{4} (o_{4}^{n}(x + 1, y) + o_{4}^{n}(x, y + 1)
+ o_{4}^{n+1}(x - 1, y) + o_{4}^{n+1}(x, y - 1) - o_{3}(x, y)) - o_{4}^{n}(x, y) \tag{7.54}
\]
Adjustment of corrective term

\[ o_4^{n+1}(x, y) = o_4^n(x, y) + \omega \text{Corrective Term}(x, y) \]  
\[ = (1 - \omega)o_4^n(x, y) - \frac{\omega}{4} o_3(x, y) \]
\[ \frac{\omega}{4} (o_4^n(x + 1), y + o_4^n(x, y - 1) + o_4^{n+1}(x - 1, y) + o_4^{n+1}(x, y - 1)) \]  

where \( \omega \) is convergence factor, \( \omega < 1 \) for under-relaxation, and \( \omega > 1 \) for over-relaxation.
Other solution for Poisson equation using discrete sine transform

- 2-D DST function defined on \( n \times n \) grid

\[
f(x, y) = 2 \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{f}(k, l) \sin \frac{\pi k x}{n+1} \sin \frac{\pi l y}{n+1}
\]

(7.57)

where \( \tilde{f}(k, l) \) is coefficients in the frequency domain.

- Looking for solution to

\[
o(x, y) = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) R'(x, y)
\]

(7.58)

\[
= R'(x - 1, y) + R'(x + 1, y) + R'(x, y - 1) + R'(x, y + 1) - 4R'(x, y)
\]

(7.59)
— Substituting with discrete sine transform

\[
\sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{\delta}(k, l) \sin \frac{x_k \pi}{n+1} \sin \frac{y_l \pi}{n+1} = \\
\sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k, l) \sin \frac{(x-1)k \pi}{n+1} \sin \frac{yl \pi}{n+1} + \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k, l) \sin \frac{(x+1)k \pi}{n+1} \sin \frac{yl \pi}{n+1} \\
- 4 \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k, l) \sin \frac{xk \pi}{n+1} \sin \frac{yl \pi}{n+1} \\
\sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k, l) \sin \frac{xk \pi}{n+1} \sin \frac{(y-1)l \pi}{n+1} + \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k, l) \sin \frac{xk \pi}{n+1} \sin \frac{(y+1)l \pi}{n+1} \\
\] (7.60)
Using

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B
\]

and, \( \sin(A - B) = \sin A \cos B - \cos A \sin B \)

one obtains as:

\[
\sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{\delta}(k,l) \sin \left( \frac{xk\pi}{n+1} \right) \sin \left( \frac{yl\pi}{n+1} \right) =
\]

\[
2 \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k,l) \sin \left( \frac{xk\pi}{n+1} \right) \cos \left( \frac{k\pi}{n+1} \right) \sin \left( \frac{yl\pi}{n+1} \right)
\]

\[
-4 \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k,l) \sin \left( \frac{xk\pi}{n+1} \right) \sin \left( \frac{yl\pi}{n+1} \right)
\]

\[
2 \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{R}'(k,l) \sin \left( \frac{xk\pi}{n+1} \right) \sin \left( \frac{yl\pi}{n+1} \right) \cos \left( \frac{l\pi}{n+1} \right)
\]

(7.61)

» Fulfilling if:

\[
\tilde{\delta}(k,l) = \left( 2 \cos \left( \frac{k\pi}{n+1} \right) + 2 \cos \left( \frac{l\pi}{n+1} \right) - 4 \right) \tilde{R}'
\]

(7.62)
Possion equation solved by using DST, and multiplying result by:

\[ 2 \cos \frac{k\pi}{n+1} + 2 \cos \frac{l\pi}{n+1} - 4 \] (7.62)
Fig. 7.12 A network of resistors can be used to perform the last step of the algorithm, i.e. the integration. Each point of the grid is connected to its immediate neighbors by a resistor of resistance R. It is also connected to its diagonal neighbors by a resistor of resistance 4R.
- Current at each grid point sums to zero:

\[ \sum_{i=1}^{n} I_i = 0 \]  \hspace{1cm} (7.64)

\[ \sum_{i=1}^{n} \left( \frac{V_i - \bar{V}}{R_i} \right) = 0 \]  \hspace{1cm} (7.65)

» Considering inject current \( I \) at center point

\[ \sum_{i=1}^{n} \left( \frac{V_i - \bar{V}}{R_i} \right) + I = 0 \]  \hspace{1cm} (7.66)

» Solving \( \bar{V} \)

\[ \bar{V} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}} \left( \sum_{i=1}^{n} \frac{V_i}{R_i} + I \right) \]  \hspace{1cm} (7.67)
» Considering neighbors resistor as $R$ and diagonal neighbors resistor as $4R$

$$\sum_{i=1}^{n} R_i = \left(4 \frac{1}{R} + 4 \frac{1}{4R}\right) = \frac{5}{R} \quad (7.68)$$

» Injecting current $I = -\frac{1}{4R} o_3(x, y)$

$$\bar{V} = \frac{R}{5} \left(\frac{V(x+1, y)}{R} + \frac{V(x, y+1)}{R} + \frac{V(x-1, y)}{R} + \frac{V(x, y-1)}{R}\right) + \frac{R}{5} \left(\frac{V(x+1, y+1)}{4R} + \frac{V(x-1, y+1)}{4R} + \frac{V(x-1, y-1)}{4R} + \frac{V(x+1, y-1)}{4R}\right) - \frac{R}{5} \frac{1}{4R} o_3(x, y) \quad (7.69)$$

» Equality to described earlier

$$\bar{V} = \frac{1}{5} \left(V(x+1, y) + V(x, y+1) + V(x-1, y) + V(x, y-1)\right)$$

$$+ \frac{1}{20} \left(V(x+1, y+1) + V(x-1, y+1) + V(x-1, y-1) + V(x+1, y-1)\right) - \frac{1}{20} o_3(x, y) \quad (7.70)$$
- Resulting
  - Output image for Horn’s algorithm

**Fig. 7.13** Output images for Horn’s algorithm. The threshold was set to zero.
• Resulting and progressing in horn’s algorithm

**Fig. 7.14** The first image is the input image. The second image shows the output of the Laplacian. The third image shows the thresholded Laplacian. Additionally, 1% of the pixels has been set to zero. The integrated output image is shown on the right. Horn’s algorithm relies on the exact separation between changes of the reflectance and changes of the illuminant.