Computational Color Technology

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Chapter 5. CIE Color Spaces

5.1 CIE 1931 Chromaticity Coordinates

- Chromaticity coordinates
  - Normalized tristimulus values X, Y, and Z
  - Projection of tristimulus space to 2D *x*-*y* plane

$$x = X/(X+Y+Z),$$
$$y = Y/(X+Y+Z),$$
$$z = Z/(X+Y+Z),$$
$$and \quad x+y+z=1,$$

  - x, y, and z are the chromaticity coordinates

5.1.1 Color gamut boundary of CIEXYZ

- Gamut boundary
  - Computed from color-matching functions

$$x(\lambda) = \bar{x}(\lambda) / \left[\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda) \right]$$
$$y(\lambda) = \bar{y}(\lambda) / \left[\bar{x}(\lambda) + \bar{y}(\lambda) + \bar{z}(\lambda) \right]$$

    - Using only two chromaticity coordinates x and y
      - Three chromaticity coordinates sum to 1
  - Gamut boundary
    - CIE1931(2°) and 1964(10 °) CMF
    - Gamut from CIE 1931 is larger
      - In green, blue and purples regions

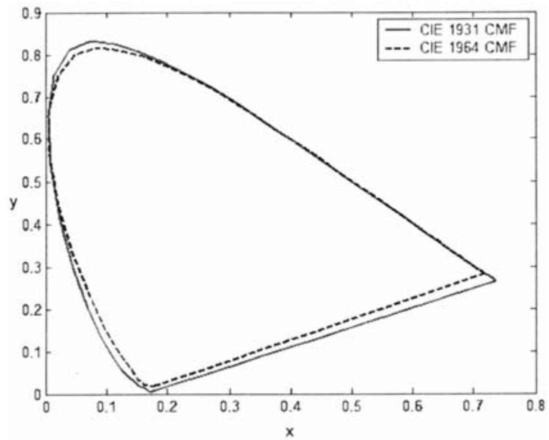

Figure 5.1 Color gamuts of CIEXYZ space.

◆ Chromaticity diagram

- Dominant wavelength
  - Based on Grassmann's laws
    - Chromaticities of all additive stimulus mixtures from two primary colorants lie along a straight line in the chromaticity diagram
  - Dominant wavelength of color
    - Obtained by extending the line connecting the color and the illuminant to the spectrum locus.
  - Complement of a spectral color
    - On the opposite side of the line connecting the color and the illuminant used
    - Color and its complement, when added together in a proper proportion, yield white.

- Dominant wavelength on purple line
  - No dominant wavelength in visible spectrum(380~770nm)
  - Specifying by the complementary spectral color with a suffix c

– Purity
  - CIE defined as the ration of two distances
    - The distance from the illuminant to the color
    - The distance from the illuminant through the color to the spectrum locus or the purple line
  - Pure spectral colors lie on the spectrum locus
    - indicating a fully saturated purity of 100%
  - The illuminant represents a fully diluted color
    - with a purity of 0%

5.2 CIELUV space

- CIEXYZ
  - Visually nonuniform color space
    - Different degrees of error distributions depending on the location
- CIELUV
  - Visually uniform color space
  - Transform of the 1976 UCS chromaticity coordinate u', v', and Y

$$L^* = 116(Y/Y_n)^{1/3} - 16 \quad \text{if } Y/Y_n \geq 0.008856, \tag{5.3a}$$

$$L^* = 903.3(Y/Y_n) \quad \text{if } Y/Y_n < 0.008856, \tag{5.3b}$$

$$u^* = 13L^*(u' - u'_n), \tag{5.3c}$$

$$v^* = 13L^*(v' - v'_n), \tag{5.3d}$$

– continued

$$u' = 4X/(X + 15Y + 3Z), \quad (5.4a)$$

$$v' = 9Y/(X + 15Y + 3Z). \quad (5.4b)$$

$$\Delta E_{uv} = \left(\Delta L^{*2} + \Delta u^{*2} + \Delta v^{*2}\right)^{1/2}. \quad (5.5)$$

$$C^*_{uv} = \left(u^{*2} + v^{*2}\right)^{1/2}, \quad (5.6)$$

$$S^*_{uv} = 13\left(u^{*2} + v^{*2}\right)^{1/2} \quad (5.7a)$$

$$S^*_{uv} = C^*_{uv}/L^*. \quad (5.7b)$$

$$h^*_{uv} = \tan^{-1}(v^*/u^*). \quad (5.8)$$

$$\Delta h^*_{uv} = \left(\Delta E^{*2}_{uv} - \Delta L^{*2} - \Delta C^{*2}_{uv}\right)^{1/2}. \quad (5.9)$$

5.2.1 Color gamut boundary of CIELUV

◆ Gamut boundary of CIE LUV

$$u'(\lambda) = 4\bar{x}(\lambda) / \left[\bar{x}(\lambda) + 15\bar{y}(\lambda) + 3\bar{z}(\lambda)\right]$$
$$v'(\lambda) = 9\bar{y}(\lambda) / \left[\bar{x}(\lambda) + 15\bar{y}(\lambda) + 3\bar{z}(\lambda)\right]$$

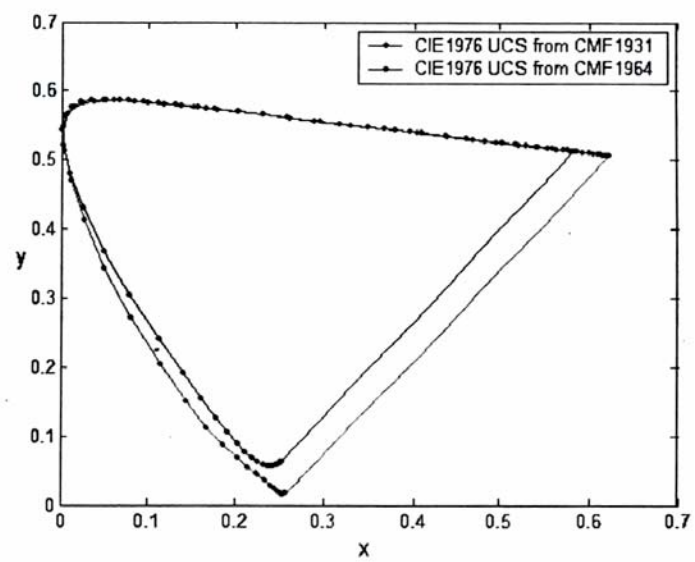

Figure 5.2 Color gamuts of CIELUV space.

5.3 CIELAB space

◆ CIELAB is a nonlinear transform of 1931 CIEXYZ

$$L^* = 116 f(Y/Y_n) - 16 \qquad \text{or}$$
$$a^* = 500[f(X/X_n) - f(Y/Y_n)]$$
$$b^* = 200[f(Y/Y_n) - f(Z/Z_n)]$$
$$\begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} = \begin{bmatrix} 0 & 116 & 0 & -16 \\ 500 & -500 & 0 & 0 \\ 0 & 200 & -200 & 0 \end{bmatrix} \begin{bmatrix} f(X/X_n) \\ f(Y/Y_n) \\ f(Z/Z_n) \\ 1 \end{bmatrix} \tag{5.11a}$$

$$f(t) = t^{1/3} \qquad 1 \geq t > 0.008856, \tag{5.11b}$$
$$= 7.787t + (16/116) \qquad 0 \leq t \leq 0.008856, \tag{5.11c}$$

$$C^*_{ab} = (a^{*2} + b^{*2})^{1/2}. \tag{5.12}$$

$$h^*_{ab} = \tan^{-1}(b^*/a^*). \tag{5.13}$$

$$\Delta E_{ab} = (\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2})^{1/2}. \tag{5.14}$$

5.3.1 CIELAB to CIEXYZ transform

◆ Inverse transform from CIELAB to CIEXYZ

$$X = X_n[a^*/500 + (L^* + 16)/116]^3 \quad \text{if } L^* > 7.9996, \quad (5.15a)$$

$$= X_n(a^*/500 + L^*/116)/7.787 \quad \text{if } L^* \le 7.9996, \quad (5.15b)$$

$$Y = Y_n[(L^* + 16)/116]^3 \quad \text{if } L^* > 7.9996, \quad (5.15c)$$

$$= Y_n L^*/(116 \times 7.787) \quad \text{if } L^* \le 7.9996, \quad (5.15d)$$

$$Z = Z_n[-b^*/200 + (L^* + 16)/116]^3 \quad \text{if } L^* > 7.9996, \quad (5.15e)$$

$$= Z_n(-b^*/200 + L^*/116)/7.787 \quad \text{if } L^* \le 7.9996. \quad (5.15f)$$

5.3.2 Color gamut boundary of CIELAB

- Gamut boundary of CIELAB
  - No chromaticity diagram, Bartleson
    - Because values of a* and b* depend on L*
  - Graphic outer boundary of the CIELAB space, Judd and Wyszecki
    - Detail is not revealed on how this graph was generated
    - Use of optimal color stimuli which is imaginary stimulus
  - Boundary of CIELAB
    - Obtained by using spectral and optimal stimuli
      - If the standard observer and illuminant are selected

– Computing the gamut boundary
  - Compute tristimulus values of the spectral stimuli and converted to L*, a*, and b*
  - Tristimulus values of the object
    - Replaced by CMF at a given wavelength
      - » Tristimulus value of the illuminant are normalized to $Y_n=1$

$$L^* = 116[\bar{y}(\lambda)/Y_n]^{1/3} - 16, \tag{5.16a}$$

$$a^* = 500\{[\bar{x}(\lambda)/X_n]^{1/3} - [\bar{y}(\lambda)/Y_n]^{1/3}\}, \tag{5.16b}$$

$$b^* = 200\{[\bar{y}(\lambda)/Y_n]^{1/3} - [\bar{z}(\lambda)/Z_n]^{1/3}\}, \tag{5.16c}$$

$$\begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} = \begin{bmatrix} 0 & 116 & 0 & -16 \\ 500 & -500 & 0 & 0 \\ 0 & 200 & -200 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda)/X_n \\ \bar{y}(\lambda)/Y_n \\ \bar{z}(\lambda)/Z_n \\ 1 \end{bmatrix}. \tag{5.16d}$$

- Two–dimensional projection
  - Concave area in the positive a*
    - From red through magenta to purple
    - Because stimuli are weak at both ends of the CMF, having very small brightness and colorfulness

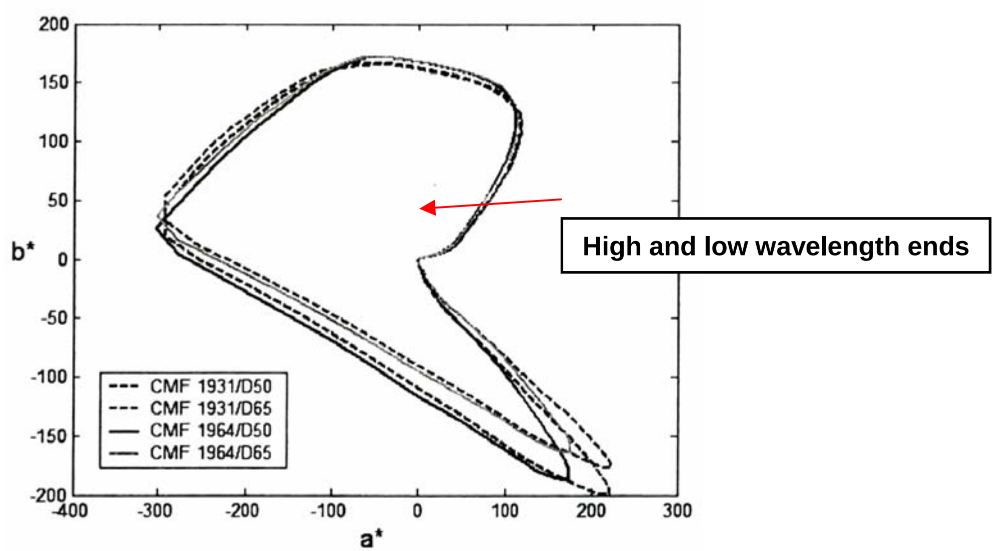

Figure 5.3 The spectral stimuli of CIE1931 and 1964 CMFs in CIELAB space under D50 or D65 illuminant.

- Saturated magenta color
  - Single spectral stimulus does not give saturated magenta colors
    - Requiring two spectral stimuli from red region and blue region
  - Obtaining the optimal two spectral stimuli
    - Fixed one stimulus and moving the other stimulus across the whole spectrum at a 5-nm interval
    - Employ the Grassmann's additivity law

$$
\begin{aligned}
X &= \left[\bar{x}(\lambda_1) + \bar{x}(\lambda_2)\right]/2, \\
Y &= \left[\bar{y}(\lambda_1) + \bar{y}(\lambda_2)\right]/2, \\
Z &= \left[\bar{z}(\lambda_1) + \bar{z}(\lambda_2)\right]/2.
\end{aligned}
$$

- Filled the concave area with additional data
- Color gamut difference
  - Small with respect to the difference in standard observer
  - Large with respect to the difference in illuminant used

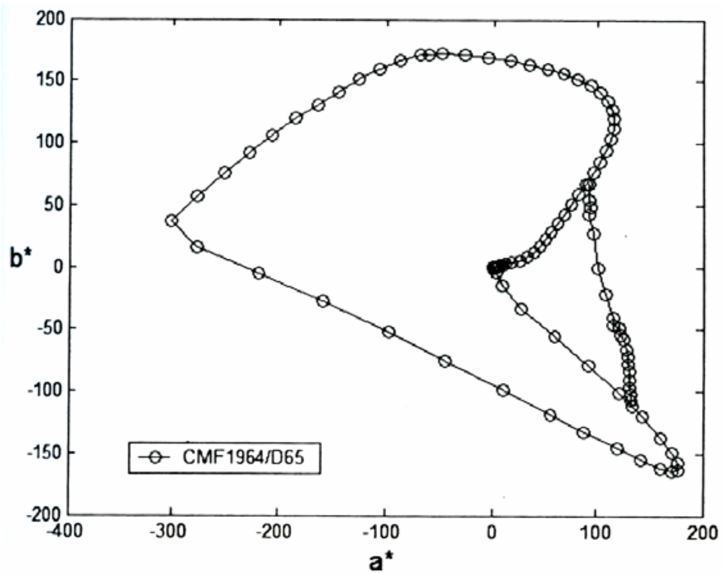

Figure 5.4 **The additional space of CIELAB.**

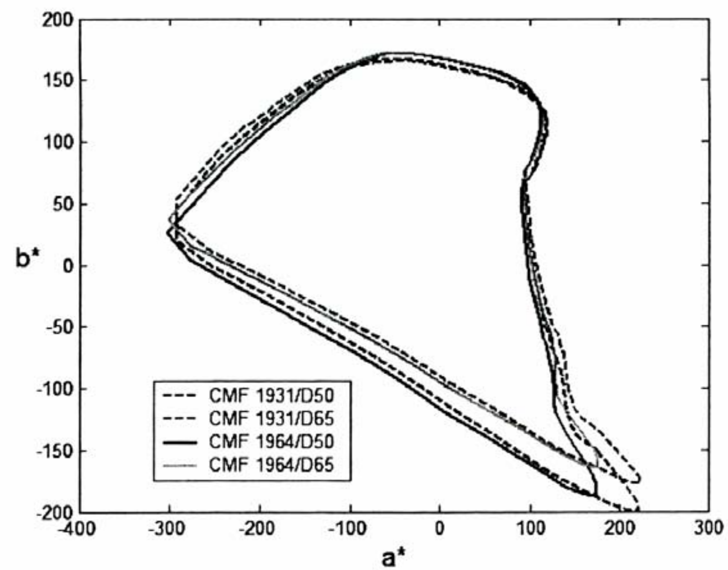

Figure 5.5 **Color gamut boundaries of CIELAB space under D50 or D65 illuminant.**

5.4 Modifications

- Uniformity and color difference measure
  - Munsell system
    - Munsell samples defined by three uniform scales of hue, value, and chroma
    - Measure to check and compare the uniformity in CIE color space

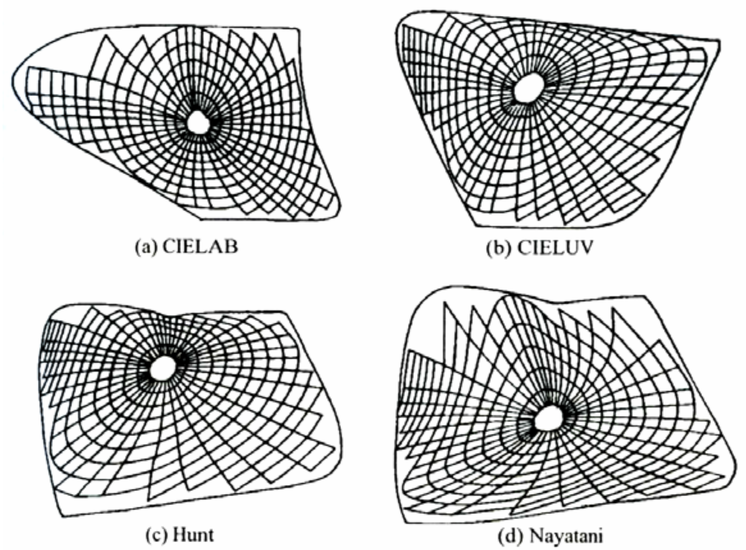

Figure 5.6 Munsell colors of value 5 in (a) CIELAB, (b) CIELUV, (c) Hunt, and (d) Nayatani spaces.

– MacAam ellipses

- Basis for a number of color difference formulas and a uniformity measure of the color space

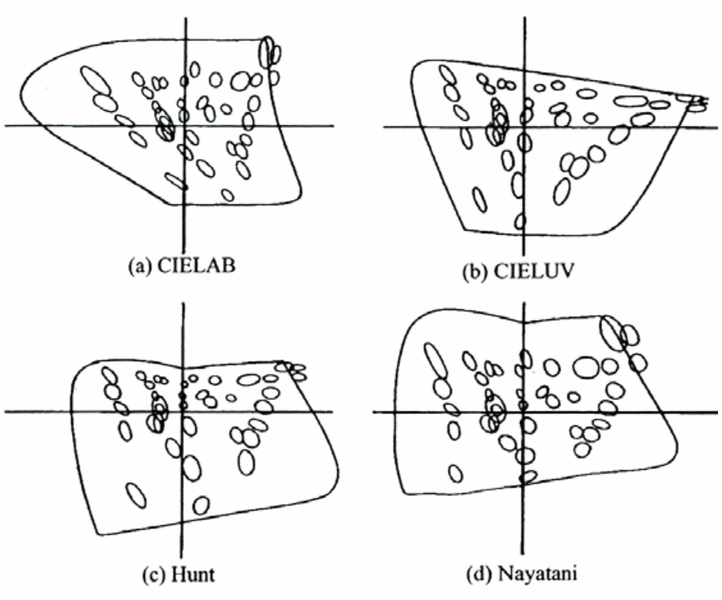

Figure 5.7 **MacAdam ellipses in the constant lightness plane with luminance factor 0.2 plotted in (a) CIELAB, (b) CIELUV, (c) Hunt, and (d) Nayatani spaces.**

– Color difference formulars

- CIE94

$$\Delta E = \left\{ [\Delta L^*/(K_L S_L)]^2 + [\Delta C^*/(K_C S_C)]^2 + [\Delta H^*/(K_H S_H)]^2 \right\}^{1/2}, \quad (5.18)$$

  – Viewing parameters, K, and scaling factor S
  – $K_L=K_C=K_H=1$, $S_L=1$, $S_C=1+0.045C^*_{ab}$, and $S_H=1+0.015C^*_{ab}$

- CIEDE2000

$$\Delta E = \left\{ [\Delta L^*/(K_L S_L)]^2 + [\Delta C^*/(K_C S_C)]^2 + [\Delta H^*/(K_H S_H)]^2 + R_T \Phi(\Delta C^* \Delta H^*) \right\}^{1/2}. \quad (5.19)$$

  – RT is introduced to deal with interactions between hue and chroma difference