Abstract

A method for generating of reflectance spectral from camera signals

- First step
  - Estimation of the mapping matrix using training set
    - Colour signals and reflectance
- Second step
  - Construction of a new reflectance function
    - Constrained least squares method
1. Introduction

- Conventional methods
  - Wiener and HF (Hasegawa and Fairchild)
    \[ f = Mc \]
    \[ c^T = (RBG) \quad f^T = (XYZ) \]  \(1\)
    where \( M \) is a 3 \( \times \) 3 matrix
  - Another approach
    - We need the information of camera sensors and illuminant
      \[ c = Qr \]  \(2\)
      where \( Q \) is a 3 \( \times \) \( n \) matrix which is dependant on the spectral power distribution of the light for illuminating object and three CCD sensors of the camera
      \[ r \approx Q^+c \]
◆ Steps of conventional methods
  – First
    • Estimation of $M$ or $Q$
  – Second
    • Reconstruction of the spectral reflectance function
  – Problem
    • Each step will inevitably introduce error (specially, second step).

◆ Motivation of proposed method
  – Directly to build a matrix $W$ based on training data set

\[ r = Wv(c) \]  

where $v(c)$ is a function of the camera response $c$. 
The New Method

- The proposed methods includes three steps.
  - Definition of the vector function $\nu(c)$.
  - Derivation of the matrix $W$.
  - Calculation of the spectral reflectance function.
The vector function $v(c)$ is defined by the polynomial equation.

$$v_0(c) = c, \quad v_1(c) = \begin{pmatrix} 1 \\ c \end{pmatrix}, \quad v_k(c) = \begin{pmatrix} v_{k-1}(c) \\ u_k \end{pmatrix},$$

where $u_k$ is a column vector and each element of it has the form of $R^{i_1}G^{i_2}B^{i_3}$, $(j_1 + j_2 + j_3 = k)$

Note
- Camera response R,G,B signals must be scaled within the range of zero and one before the calculation of the vector function $v(c)$.
Derivation of the mapping matrix $W$

- Suppose that there are $p$ colour patches

\[
S = \begin{bmatrix} r^{(1)}, r^{(2)}, \Lambda, r^{(p)} \end{bmatrix}, \quad V = \begin{bmatrix} v_k(c^{(1)}), v_k(c^{(2)}), \Lambda, v_k(c^{(p)}) \end{bmatrix}
\]  

(5)

- Then, the matrix $W$ should satisfy

\[
S = WV
\]

(6)

- Vector operator, $vec$
  
  - For example $S$

\[
\left[vec(S)\right]^T = \left((r^{(1)})^T, (r^{(2)})^T, \Lambda, (r^{(p)})^T\right)
\]

(7)

- Thus, if we let

\[
s = vec(S), \quad w = vec(W), \quad A = V^T \otimes I_n
\]

(8)

here, operator $\otimes$ is the kronecker product operator

$I_n$ is the identity matrix of size $n$
**Kronecker Product and the $\text{vec}$ Operator**

**Definition 1.** Let $A$ be an $n \times p$ matrix and $B$ an $m \times q$ matrix. The $mn \times pq$ matrix

$$A \otimes B = \begin{bmatrix}
    a_{1,1}B & a_{1,2}B & \cdots & a_{1,p}B \\
    a_{2,1}B & a_{2,2}B & \cdots & a_{2,p}B \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n,1}B & a_{n,2}B & \cdots & a_{n,p}B
\end{bmatrix}$$

**Definition 2.** The $\text{vec}$ operator creates a column vector from a matrix $A$ by stacking the column vectors of $A = [a_1 \ a_2 \ \cdots \ a_n]$ below one another:

$$\text{vec}(A) = \begin{bmatrix}
    a_1 \\
    a_2 \\
    \vdots \\
    a_n
\end{bmatrix}.$$
Then, it can be shown from Eq. (6) that

\[ Aw = s \]  

(9)

- The linear system of equation (9) may have no solution in normal sense, but it always has a least squares solution.

- Some constraints
  - Black and white patches

\[ S_E = [r_b, r_w], V_E = [v_k(c_b), v_k(c_w)] \]

\[ s_E = vec(S_E), A_E = V_E^T \otimes I_n \]  

(10)

\[ A_E w = S_e \]  

(11)

- \( w \) must not be less than \( b_L \) and must not be greater than \( b_U \)
The Constrained Least Squares Problem for Finding Matrix $W$

\[
\begin{align*}
\text{Min} & \quad \|Aw - s\|^2 \\
\text{Subject to :} & \quad A_E w = s_E, \quad b_L \leq w \leq b_U \\
\text{Note} & \quad \|x - y\| \text{ is the Euclidian distance of the two vectors } x \text{ and } y \\
\text{The resultant spectral reflectance function may include some values outside the range of 0 and 1}
\end{align*}
\]
The Constrained Least Squares Problem for Reconstructing Spectral Reflectance Function

\[
\text{Min} \frac{1}{v} \| v - v_k(c) \|^2
\]

- Subject to: \( 0 \leq Wv \leq 1 \)

- Note that 0 and 1 here represent vectors with all elements being zero and one respectively.
The simulated data
- Munsell 1560 samples
- Textile 705 samples
- GretagMacbeth ColorChecker Digital Chart (228 form 240 samples)
- Multiplicative Gaussian noise

\[
\begin{bmatrix}
err_{\text{noise}}
\end{bmatrix}^T = \varepsilon(\xi_1 R, \xi_2 G, \xi_3 B), \text{ with } \varepsilon = 0.01
\]  

- Training sets
  - One every 10 samples (Munsell)
  - One third samples (Textile)
  - One-half samples (DC data)
◆ Estimation of weight matrix $Q$ (Wiener and HF methods)

$$\left\| \begin{pmatrix} R^f \\ G^f \\ B^f \end{pmatrix} - Qr \right\|^2$$

Fig. 1. Sensitivity Function of the Camera

Fig. 2. Sensors of the Camera
The measure of performance

\[
err_r = \sqrt{\frac{1}{n} \| r_j - \tilde{r} \|^2} = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (r_j - \tilde{r}_j)^2}
\]  

(13)

Fig. 3. The Estimated Camera Weight
Table 1. Performance of each method based on the testing data sets in terms of median (Med) and maximum (Max) of spectra difference using the simulated data

<table>
<thead>
<tr>
<th></th>
<th>Proposed Method</th>
<th></th>
<th>Wiener Method</th>
<th></th>
<th>HF Method</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Med</td>
<td>Max</td>
<td>Med</td>
<td>Max</td>
<td>Med</td>
<td>Max</td>
</tr>
<tr>
<td>Munsell</td>
<td>0.015</td>
<td>0.116</td>
<td>0.020</td>
<td>0.136</td>
<td>0.031</td>
<td>0.089</td>
</tr>
<tr>
<td>Textile</td>
<td>0.021</td>
<td>0.099</td>
<td>0.035</td>
<td>0.115</td>
<td>0.032</td>
<td>0.119</td>
</tr>
<tr>
<td>DC</td>
<td>0.011</td>
<td>0.083</td>
<td>0.018</td>
<td>0.140</td>
<td>0.026</td>
<td>0.086</td>
</tr>
</tbody>
</table>

Table 2. Performance of each method based on the testing data sets in terms of median (Med) and maximum (Max) of CIELAB colour difference using the simulated data

<table>
<thead>
<tr>
<th></th>
<th>Proposed Method</th>
<th></th>
<th>Wiener Method</th>
<th></th>
<th>HF Method</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Med</td>
<td>Max</td>
<td>Med</td>
<td>Max</td>
<td>Med</td>
<td>Max</td>
</tr>
<tr>
<td>Munsell</td>
<td>2.30</td>
<td>15.14</td>
<td>2.60</td>
<td>28.77</td>
<td>3.27</td>
<td>10.25</td>
</tr>
<tr>
<td>Textile</td>
<td>2.38</td>
<td>12.01</td>
<td>3.46</td>
<td>33.78</td>
<td>2.80</td>
<td>11.39</td>
</tr>
<tr>
<td>DC</td>
<td>1.72</td>
<td>10.96</td>
<td>2.61</td>
<td>10.97</td>
<td>2.77</td>
<td>7.65</td>
</tr>
</tbody>
</table>

Table 2. Performance of each method based on the testing data sets in terms of median (Med) and maximum (Max) of spectral difference and colour difference respectively using the simulated data

<table>
<thead>
<tr>
<th></th>
<th>Proposed Method</th>
<th></th>
<th>Wiener Method</th>
<th></th>
<th>HF Method</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Med</td>
<td>Max</td>
<td>Med</td>
<td>Max</td>
<td>Med</td>
<td>Max</td>
</tr>
<tr>
<td>$err_r$</td>
<td>0.011</td>
<td>0.120</td>
<td>0.051</td>
<td>0.161</td>
<td>0.052</td>
<td>0.165</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>1.38</td>
<td>9.82</td>
<td>13.07</td>
<td>52.05</td>
<td>13.52</td>
<td>41.19</td>
</tr>
</tbody>
</table>
Fig. 4. A real (curve without marking) and generated reflectance functions by proposed (marked “*”), Wiener (marked “o”), and HF (marked “+”) methods.
Conclusion

◆ A new method for generating spectral reflectance functions based on signals
  – The method characterizes the camera and estimate the matrix $W$
    • To map the camera’s signal to its reflectance function
  – Constrained least squares problem is used
    • Some constraints
  – It does not need to know or to estimate the camera’s sensors