Subpixel Shift Estimation Method for the Registration of Noisy Images Using a Wiener Filtered Local Region

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Abstract we propose an accurate subpixel shift estimation method for the registration of noisy images. The proposed method uses a specific local region to perform phase correlation instead of the whole noisy image and applies an adaptive Wiener filter to a local region for suppressing noises. Firstly, a local region is selected in the vicinity of the feature points from Harris corner detection by analyzing gradient and distribution of frequency component. Next, adaptive Wiener filter which is varying the size of window depending on the property of a local region is applied to eliminate noises. And then, iterative phase correlation is performed in a restricted iteration range with coarse to fine method. In the experiments, the proposed method shows higher accuracy in registering noisy images than other methods.

1 Introduction

Image registration is the process of aligning two or more images of the same scene taken from slightly different viewpoints. Image registration is used in many applications of image processing, including medical imaging, remote sensing, computer vision, and super-resolution[1]. It is important to estimate the shift between images accurately in image registration since the resulting image is directly affected by the precision of the estimated shift. Especially, in high quality image registration or super-resolution imaging, the shift has to be estimated with subpixel accuracy.

A commonly used subpixel shift estimation method is phase correlation which is based on the Fourier shift property. In phase correlation, the shift between two images in the spatial domain is transformed into a linear phase difference in the frequency domain [2-4]. The subpixel shift is then obtained using a fitting function in the vicinity of the maximum peak of the inverse Fourier transform of the normalized cross-power spectrum [5-6]. Another approach to estimate the subpixel shift is to find the best fit phase plane of the normalized cross-power spectrum in the frequency domain. The slope of the plane is used to estimate the subpixel shift. But it is inaccurate for noisy images. Thus, to overcome this problem, recently several methods have been proposed. A correlation masking operator has been suggested for attenuating additive white Gaussian noise. Correlation masking operator method used the fact that most of the correlation energy is concentrated in a small area in frequency domain [7]. Meanwhile, a high order statistics technique has been proposed to address the problem of subpixel registration in noisy images, where the characteristics of a bispectrum are utilized to suppress additive white Gaussian noise [8]. Particularly, it is useful to register images under low SNR environments or in the presence of cross-correlated channel noise. In addition, esinc function that is a modified sinc function has been proposed to a fitting function. Esinc function consists of applying a Gaussian weighting to a sinc function, making it better tuned to approximate the phase correlation of noisy images [6].

In this paper, we propose an accurate subpixel shift estimation to address precise registration between noisy images. The process of the proposed method
consists of two parts. One is selecting a local region to perform phase correlation. Within an image, a local region is selected that is both relatively less affected by noises and includes an intensity variation. And adaptive Wiener filter which is varying the size of window depending on the property of a local region is applied to eliminate additive white Gaussian noise. The other is iterative phase correlation to calculate subpixel shift. Phase correlation which is invariant to blur is performed iteratively in restricted range with coarse to fine method.

2 Problem in conventional method

Phase correlation is an efficient technique for estimating the shift between images. However, if the image is affected by noises such as additive white Gaussian noise and aliasing artifacts, the values of phase correlation which is the inverse Fourier transform of the normalized cross power spectrum are distorted. Thus, the estimation error is largely increased. The noisy image and its inverse Fourier transform of the normalized cross power spectrum are given in figure 1. As seen in figure 1 (b), the inverse Fourier transform of the normalized cross power spectrum is highly affected by noises. To estimate subpixel shift accurately in noisy image, the effects due to noises have to be reduced in phase correlation.

3 Proposed method

The proposed method is depicted in figure 2. It can be divided into two parts. In the first part, a proper local region is selected to perform phase correlation. This process includes Harris corner detection, gradient of the image, frequency analysis, and adaptive Wiener filtering. The other part involves the iterative process to calculate subpixel shift, including restriction of iteration range and phase correlation that is invariant to blurring artifacts[9].

3.1 Selecting candidates of a local region

The selected candidates of a local region can replace the whole image to estimate subpixel shift. Therefore, candidates of a selected local region have to be satisfied to several conditions.

First, a local region contains edges, where an edge means the intensity variation. However, since an edge only has one directional shift information, corners that are cross points of various directional edges have to be detected. For example, vertical shift is not estimated in horizontal edge. In other word, according to direction of the intensity variation, a limited directional shift is estimated. In proposed method, Harris corner detection algorithm is used to detected corners. However, Harris corner detection is significantly affected by additive white Gaussian noise. For example, in Harris corner detection, additive white Gaussian noise in the flat region of the image is regarded as a corner. To overcome this defect, gradient of image using canny edge detector has to be calculated. Since canny edge detector is independent to additive white Gaussian noise, the magnitude of gradient at selected corner point in flat region which is a fault corner due to additive white Gaussian noise is small. Thus, as comparing magnitude of gradient, fault corners can be rejected.

Figure 1. (a) Noisy image, (b) inverse Fourier transform of the normalized cross power spectrum

![Flow of the proposed method](image.png)
3.2 Frequency analysis
The candidates of a local region are analyzed as regards distribution of frequency components. Thus, the local regions are transformed to frequency domain. And then, the rectangular coordinates in frequency domain are transformed into polar coordinates for the convenience of analysis. Local region containing high frequency is apt to be easily affected by aliasing artifacts and additive white Gaussian noise. Thus, a local region containing high frequency components should be rejected from the candidate of a local region. And low frequency in a local region that is a common and redundancy component is also rejected. As a result, a specific local region that contains corner, large magnitude of gradient and middle frequency components is selected. According to Eq. (1), a proper local region that has the maximum K value is selected to the final local region.

\[
K = \int_{r=0}^{r=2\pi} \int_{\theta=0}^{\theta=\pi} |F(r, \theta)| \, d\theta \, dr
\]  

(1)

where \(r\) is the radius, \(\theta\) is the angle, and \(F\) is the Fourier transform of the selected local region.

3.3 Wiener filtering of a selected local region
Although a local region has been selected, there is still additive white Gaussian noise. To more accurately estimate subpixel shift between images, additive white Gaussian noise in a local image has to be suppressed. As possible as information of the original image is preserved, only additive white Gaussian noise has to be suppressed effectively. Wiener filter which is used in proposed method defined as Eq. (2).

\[
p_w(x, y) = \frac{\sigma_f^2(x, y)}{\sigma_f^2(x, y) + \sigma_n^2} \left[ g(x, y) - m_f(x, y) \right] + m_f(x, y)
\]

(2)

where \(\sigma_f^2\), \(m_f\) is the variance and mean of original image, \(\sigma_n^2\) is the variance of noise, \(g\) is the pixel value of degraded image, and \(p_w\) is the Wiener filtered pixel value. However, since the size of window of conventional Wiener filter is fixed, it induces some undesired artifacts such as blocking and blurring. The proposed method varies the size of window according to the flatness of a local region. The size of window is enlarged when a local region is determined to a flat local region. Otherwise the size of window is reduced. To determine the flatness of a local region, Eq. (3) is used. A local region is divided into sublocal region. And averaging the absolute difference of each subdivided local region.

\[
T = \frac{1}{L} \sum_{k=0}^{L-1} \sum_{j=0}^{N-1} \left| \left( p(i + c_k, j + c_k) - p(i, j) \right) \right|
\]

(3)

where \(P\) is the pixel value, \(L\) is the number of subdivided local region, \(c_k\) is a center of subdivided local region, and \(M, N\) is the size of subdivided local region. According to flatness of a local region, the size of window is determined by Eq. (4)

\[
n = \begin{cases} 3, & T \geq 23 \\ 5, & T < 23 \end{cases}
\]

(4)

where \(n\) is the window size and \(T\) is the flatness.

3.4 Iterative process
After performing phase correlation using a local region, subpixel shift is calculated. In proposed method, iterative phase correlation is used rather than a fitting function such as sinc, esinc function. When applying these functions to fit a phase correlation to calculate subpixel shift, relative estimation error is generated. As the selected local region is so small, the assumption in conventional method that the size of the image is much larger than the downsampling factor is not valid. To avoid these estimation errors, the proposed method does not use a fitting function but performs phase correlation iteratively. It is applicable subpixel estimation method to small-sized images.

Iterative phase correlation is simple but has high computation. To reduce computation, the number of iteration is reduced by restricting iteration range and coarse to fine method. The range of iteration is restricted by analyzing the maximum peak and the two adjacent values of the function of phase.
correlation, which is the inverse Fourier transform of the normalized cross power spectrum. If the condition of two adjacent values of peak is satisfied by Eq. (5), the range of the iterative phase correlation can be limited within 0.5 pixels. After determining the range of iteration, coarse to fine method is applied.

\[
d = \begin{cases} 
  c_i < d < c_i + 0.5 & \text{if } f(c_{i-1}) < f(c_{i+1}) \\ 
  c_i - 0.5 < d < c_i & \text{if } f(c_{i-1}) > f(c_{i+1}) 
\end{cases} 
\]  

(5)

where \( c_i \) is the maximum peak point of the inverse Fourier transform of the normalized cross power spectrum, \( c_{i-1}, c_{i+1} \) are the two adjacent points of the maximum peak, \( f \) is the inverse Fourier transform of the normalized cross power spectrum, and \( d \) is the range of the iteration process. For example, the parameters are determined to \( c_i = 2, c_{i-1} = 1, \) and \( c_{i+1} = 3 \) in figure 3 (a). The magnitude of \( c_{i+1} \) is larger than \( c_{i-1} \). Thus, the range of iteration, \( d \), becomes \( 2 < d < 2 + 0.5 \) from Eq. (5). As the original shifted value is 2.25 pixels, the range of iteration by Eq. (5) is correct. Eq. (5) is also reasonable in noisy images. When comparing figures 3 (a) and (c), the inequality of magnitude of two adjacent values is not changed. Eq. (5) is still possible.

4 Experiments

To test the performance of the proposed method, simulated subpixel shifted images were used with different levels of additive white Gaussian noise. The size of simulated image was 512 x 512 pixels, and additive white Gaussian noise is zero mean and four standard deviations from 0 to 0.01 which was normalized to 1. The subpixel shifted value is 1.5, 1.25, and 1.125, respectively. The experimental result is represented in Table 1. The figures in table 1 are average error of three estimated subpixel shift where the images are shifted 1.5, 1.25, and 1.125. Comparing the conventional methods and the proposed method, the proposed method has less average estimation error than other methods mostly.

Figure 3. The two adjacent values of the maximum peak according to subpixel shift and additive white Gaussian noise (a) noise free and 2.25 pixel shifted image (b) noise free and 2.75 pixel shifted image (c) additive white Gaussian noise (S.D.= 0.01) and 2.25 pixel shifted image (d) additive white Gaussian noise (S.D.= 0.01) and 2.75 pixel shifted image.

5 Conclusion

This paper proposes an accurate subpixel shift estimation method for registering a noisy image. To avoid effect of noises, a specific local region in image which is applied adaptive Wiener filter is used to perform phase correlation. For an accurate estimation, iterative phase correlation is then used to calculate subpixel shift in restricted range with coarse to fine method. Experimental results showed improvement in estimation accuracy of the proposed method when compared to conventional methods.

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Table 1. Average error for estimated subpixel shift in noisy image using various methods.

<table>
<thead>
<tr>
<th>S.D. of AGN</th>
<th>direction</th>
<th>parabola</th>
<th>sinc</th>
<th>esinc</th>
<th>proposed method</th>
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<td>x</td>
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<td>0.505</td>
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<td>y</td>
<td>0.291</td>
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<tr>
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<tr>
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<td>0.466</td>
<td>0.265</td>
<td>0.191</td>
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</table>

References


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