A Test based on Normal Score for Efficient Edge Detection
In Image

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ABSTRACT

The current paper proposes an efficient method for edge detection in original and noisy images using Waerden’s statistic. Edges represent a significant amount of information on an image. For example, edges reveal the location of objects, their shape and size, and something about their texture. Since edges represent where the intensity of an image moves from a low value to a high value or vice versa, edge detection is often the first step in image segmentation. As a field of image analysis, image segmentation groups pixels into regions to determine the image composition. Therefore, the current paper describes the nonparametric Wilcoxon test and parametric T test based on statistical hypothesis testing for edge detection. Here, the threshold is determined by specifying a significance level, whereas Bovik, Huang, and Munson considered a range of possible test statistic values for the threshold. In the current study, the test statistic is calculated based on pixel gray levels obtained using an edge-height parameter and compared with the threshold determined by a significance level. Experiments were conducted to evaluate the performance of these methods in both original and noisy images. As a result, the Wilcoxon and T test was found to be sensitive to a noisy image, whereas the proposed Waerden test was robust in both noisy and noise-free images. Furthermore, when compared with Sobel, LoG, and Canny operators, the proposed Waerden test was also more effective in both noisy and noise-free images.

Keywords: Edge detection, Noisy image, Wilcoxon and T test, Waerden test, Canny detector

1. INTRODUCTION

The edges in an image reveal the location of objects, their shape and size, and something about their texture. Edges also indicate where the intensity of an image moves from a low value to a high value or vice versa. In general, it is very difficult to detect an edge when only using a low frequency filter to reduce noise, as noise and edges both have a high value in a noisy image.

When performing edge detection using a differential operator, such as a Sobel, LoG, or Canny operator, edges are identified according to a threshold, thus the selection of a suitable threshold for a noisy image is difficult and results in controversial points so that edge information is lost. Therefore, statistical techniques have been developed to deal with such problems.

Bovik et al.\textsuperscript{1} describe three statistically motivated techniques for detecting edges in gray-level images. Two similar methods based on linear rank sums are described, where Wilcoxon and median statistics are implemented in a modified form and found to perform effectively in both noisy and uncontaminated sample images. In addition, a novel approach based on fitting order statistics to image data is also presented. They provide numerous examples and examine the computational requirements.

Huang and Tseng\textsuperscript{2} suggest the statistical theory of hypothesis testing to compensate for the blurring of edges with noise filtering, as edges correspond to high frequencies. They detect an edge using the maximum likelihood method for the change-point problem, yet the drawback is a high computational cost.

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Thune et al.\textsuperscript{3} propose edge detection in noisy data using a finite mixture distribution analysis, however, the edge detection deteriorates in the case of satisfying the normal assumption of data.

Lim and Park\textsuperscript{4} detect edges using statistical methods for the change-point problem, where hypothesis testing is performed on the gray levels differences to determine whether any $n \times n$ sub image contains edge segments. Their proposed edge detection method is based on the two-sample Kolmogorov-Smirnov test, along with the likelihood ratio test and Wolfe-Schechtman test for the change-point problem. In experiments to assess the performance of these methods in noisy and uncontaminated sample images, the main difficulty for application to real images was the high computational cost when using a nonparametric method.

Lim and Park\textsuperscript{5} also employ two-sample location tests, such as the Wilcoxon test and T test for detecting edges in noisy images. They compute test statistics based on pixel gray levels obtained using an edge-height parameter, then compare them with a threshold determined by a significance level. When experimental results with sample images were compared, the Wilcoxon test was found to be robust, regardless of the noise level in the image.

Accordingly, the current paper proposes an efficient method of edge detection in original and noisy images using Waerden’s statistic and compares the performance with the nonparametric Wilcoxon test and parametric T test. Using an edge-height model, the Waerden, Wilcoxon, and T test statistics are computed and the existence of an edge established using a threshold determined by a significance level. The edge detection performance is also compared with the LoG, Sobel, and Canny methods using a subjective threshold. Chapter 2 introduces the Wilcoxon test and T test in an edge-height model, while chapter 3 presents the proposed Waerden test. Chapter 4 outlines the experimental study used to assess the performance of these methods in original and noisy images. Finally, chapter 5 offers some conclusions on the image processing results given in chapter 4.

2. STATISTICAL TEST FOR EDGE DETECTION

Edge detection is essential for recognizing an object. Traditional methods of edge detection involve filtering the image then applying simple techniques to detect edges. However, noise filtering also blurs edges, since they correspond to high frequencies. Therefore, the statistical theory of hypothesis testing proposes that filtering and edge detection should both take place at the same time. Therefore, this chapter introduces Wilcoxon testing and T testing as edge detection methods that use the statistical method in the context of an edge-height model.

Consider $n \times n$ square neighborhoods, with $n$ odd, where $n = 3$, as shown in Fig. 1. The current paper only describes the detection operation for vertically oriented edges, as the final edge decision is simply taken to be the "OR" of each directional decision.

Assume the gray-level values $X_1, \cdots, X_N$ and $X_{N+1}, \cdots, X_{2N}$ correspond to the left adjacent neighborhood area $N_L$ and right adjacent neighborhood area $N_R$, respectively, and are random samples with the continuous distribution function $F_1(x) = F(x - \mu_1)$ and $F_2(x) = F(x - \mu_2)$, respectively. Here $N = n^2$ and $\mu_1, \mu_2$ are the shift parameters.

In this chapter, the gray-level values $X_i$ are modified using the edge-height parameter $\delta$ as follows:

$$A_i = \begin{cases} X_i + \delta & ; X_i \in N_L \\ X_i & ; X_i \in N_R \end{cases}$$  \hspace{1cm} (1)

$$B_i = \begin{cases} X_i - \delta & ; X_i \in N_L \\ X_i & ; X_i \in N_R \end{cases}$$  \hspace{1cm} (2)

As such, a vertical edge can be detected using the modified gray-level values and testing the two hypotheses at the same time.

(Hypothesis1)  $H_0 : \mu_2 \leq \mu_1 + \delta$ \hspace{0.5cm} vs \hspace{0.5cm} $H_1 : \mu_2 > \mu_1 + \delta$  \hspace{1cm} (3)

and

(Hypothesis2)  $H_0 : \mu_1 \leq \mu_2 + \delta$ \hspace{0.5cm} vs \hspace{0.5cm} $H_1 : \mu_1 > \mu_2 + \delta$,  \hspace{1cm} (4)
When (Hypothesis1) and (Hypothesis2) are both rejected as below the significant level $\alpha$, no edge is assumed at the significant level $\alpha$. In this case, noise or an edge less than $\delta$ is removed when $\delta$ increases based on the modified gray-level values $\{A_i\}$ and $\{B_i\}$.

### 2.1 Wilcoxon test

First, Wilcoxon's rank sum test statistic for (Hypothesis1) in equation (3) is given by

$$W_A = \sum_{i=1}^{2N} R_i I_{Ai},$$

where $R_i$ is the rank of $A_i$ and $I_{Ai}$ is defined by

$$I_{Ai} = \begin{cases} 0, & R_i \in \{A_1, \ldots, A_N\} \\ 1, & R_i \in \{A_{N+1}, \ldots, A_{2N}\} \end{cases}$$

The test statistic in equation (4) for (Hypothesis2) is given by

$$W_B = \sum_{i=1}^{2N} S_i I_{Bi},$$

where $S_i$ is the rank of $B_i$ and $I_{Bi}$ is defined by

$$I_{Bi} = \begin{cases} 1, & S_i \in \{B_1, \ldots, B_N\} \\ 0, & S_i \in \{B_{N+1}, \ldots, B_{2N}\} \end{cases}$$

Therefore, the Wilcoxon test statistic for edge detection is given by

$$W^* = \max(W_A, W_B)$$

If $N$ is large, the standardized statistic

$$Z_{w^*} = \frac{W^* - E(W^*)}{\sqrt{\text{Var}(W^*)}}$$

is the standard normal distribution under the assumption there is no edge. Here $E(W^*) = N(2N + 1)/2$, $\text{Var}(W^*) = N^2(2N + 1)/12$. If $Z_{w^*}$ is larger than $z_{\alpha}$, an edge is determined to exist at the significant level $\alpha$.

### 2.2 T Test

Let $F$ be the distribution of $N(0, \sigma^2)$. Hence (Hypothesis1) in equation (3) is equal to test the alternative hypothesis $H_1: \mu_2 \geq \mu_1$ for the null hypothesis $H_0: \mu_1 \geq \mu_2$ and the T test statistic is given by
\[ T_A = \frac{\bar{A}_R - \bar{A}_L}{S_p[2/N]^{1/2}}, \]  \hspace{1cm} (11)

where

\[ \bar{A}_L = \sum_{i=1}^{N} A_i / N, \quad \bar{A}_R = \sum_{i=N+1}^{2N} A_i / N, \]

\[ S_p^2 = \left( \sum_{i=1}^{N} (A_i - \bar{A}_L)^2 + \sum_{i=N+1}^{2N} (A_i - \bar{A}_R)^2 \right) / (2N - 2). \]  \hspace{1cm} (12)

Using the ditto method, the test statistic for (hypothesis 2) in equation (4) is given by

\[ T_B = \frac{B_R - B_L}{S_p[2/N]^{1/2}}, \]  \hspace{1cm} (13)

where

\[ \bar{B}_L = \sum_{i=1}^{N} B_i / N, \quad \bar{B}_R = \sum_{i=N+1}^{2N} B_i / N, \]

\[ S_p^2 = \left( \sum_{i=1}^{N} (B_i - \bar{B}_L)^2 + \sum_{i=N+1}^{2N} (B_i - \bar{B}_R)^2 \right) / (2N - 2). \]  \hspace{1cm} (14)

Therefore, under the null hypothesis that there is no edge, the following test statistic is used

\[ T^* = \max(T_A, T_B) \]  \hspace{1cm} (15)

\( T^* \) is distributed as a \( t \) distribution with \( 2N - 2 \) degrees of freedom under the null hypothesis. If \( T^* \) is greater than \( t_{\alpha^{-1}}(2N - 2) \), an edge is determined to exist. Here \( \alpha^* \) is \( 1 - \sqrt{1 - \alpha} \).

3. EDGE DETECTION USING PROPOSED TEST BASED ON NORMAL SCORES

Chapter 2 described the two-sample nonparametric Wilcoxon test and parametric \( t \) test. Edge detection using a statistical hypothesis test has the advantage of edge detection outwith the bandwidth. Hence, edge detection using Waerden’s rank test statistic is considered as one of nonparametric test.

In statistics, the ARE (Asymptotic Relative Efficiency) is used to show the relative efficiency of two test statistics. \(^6\) The ARE \( E(C, W) \) of Waerden’s \( C \) test relative to Wilcoxon’s \( W \) test has already been evaluated under several type variable populations, as follows:\(^7\) if \( E(C, W) \) is greater than 1, then \( C \) is more efficient than \( W \).

Therefore, the current paper proposes edge detection using Waerden’s \( C \) test, which is a two-sample nonparametric test and more efficient than Wilcoxon’s \( W \) test.

Table 1. ARE of Waerden’s C test relative to the test based on W

<table>
<thead>
<tr>
<th>( F )</th>
<th>Normal</th>
<th>Uniform</th>
<th>Logistic</th>
<th>Double Exponential</th>
<th>Cauchy</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(C, W) )</td>
<td>1.047</td>
<td>( \infty )</td>
<td>0.955</td>
<td>0.847</td>
<td>0.708</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>
Waerden’s rank sum test statistic for (Hypothesis1) in equation (3) is given by
\[ C_A = \sum_{i=1}^{2N} \Phi^{-1}\left( \frac{R_i}{2N + 1} \right) I_A, \]
where \( \Phi^{-1}(t) \) is the \( t \)-th quantile of the standard normal distribution.
Similarly, the test statistic for (Hypothesis2) in equation (4) is given by
\[ C_B = \sum_{i=1}^{2N} \Phi^{-1}\left( \frac{S_i}{2N + 1} \right) I_B, \]
Waerden’s test statistic for edge detection is given by
\[ C^* = \max(C_A, C_B) \]
If \( N \) is large, the standardized statistic
\[ Z_{c'} = \frac{C^* - E(C^*)}{\sqrt{\text{Var}(C^*)}} \]
has a normal distribution under the null hypothesis that no edge exists. Where
\[ E(C^*) = 0, \quad \text{Var}(C^*) = N \left[ \sum_{i=1}^{2N} \left( \Phi^{-1}\left( \frac{i}{2N + 1} \right) \right)^2 \right] / 2N(2N - 1) \]
and if \( Z_{c'} \) is larger than \( z_{\alpha'} \), an edge is determined to exist at the significant level \( \alpha' \). Here, \( \alpha' = 1 - \sqrt{1 - \alpha} \).

4. EXPERIMENTS AND DISCUSSION

An experiment was conducted to compare the performances of the various tests, including the Wilcoxon test, T test, and Waerden test with Sobel, LoG, and Canny operators in a plane image. An experiment was also conducted using a plane image with added Gaussian noise G10, where a 0 noise average and standard deviation of 10 were added to the original image. The width of the Gaussian noise was represented by the following signal-to-noise ratio(SNR, dB),
\[ \text{SNR} = 20 \log(25/\sigma) \]
where the assumed edge height was 25 and \( \sigma^2 \) was the noise variance.

4.1 Original image
Fig. 2 (a) and (b) show the original plane image and image with Gaussian noise G10. The proposed Waerden test, Wilcoxon test, and T test were examined with a significance of 0.0005, while the Canny method was used with a double threshold of 20 and 40. The Sobel and LoG methods were used with a threshold of 80, and the LoG method was the case of Masks 4, 5, and 7. Fig. 3 represents a histogram of the original plane image.

4.2 Gradient operators
Fig. 4 and 5 show the edge maps obtained by the Sobel, LoG, and Canny methods from figs 2 (a) and (b). Fig. 4, obtained from fig 2 (a), shows that the Sobel and Canny methods detected the edges better than the LoG method. The edges obtained by the Canny method were the thinnest among the three results, due to the non-maximum suppression of the Canny method and double threshold. Fig. 5, obtained from fig 2(b), shows that the efficiency of the edge detection by the three conventional methods was no better than that of the proposed Waerden test.
4.3 Statistical edge detection

Edge detection using the statistical theory of hypothesis testing is commonly used as filtering and edge detection both take place at the same time.

Based on figs. 2 (a) and (b), fig. 6 and 7 show the edge maps resulting from the proposed Waerden test and Wilcoxon and T test when the edge height $\delta = 10, 20$. In both cases, the edge detection by the Waerden test was superior to that by the Wilcoxon and T test.

Accordingly, the Waerden test was more effective than the other statistical tests in both noisy and noise-free images when $\alpha = 0.0005$. Furthermore, the Waerden test was more insensitive to noise than the Sobel, LoG, and Canny operators in the noisy image.
Figure 4: Edge Map in Fig 2(a) (a) Sobel operator, (b) Canny operator, (c) LoG operator with Mask 5

Figure 5: Edge Map in Fig 2(b) (a) Sobel operator, (b) Canny operator,
Figure 6: Edge Map with delta=20 in Fig 2(a) (a) The proposed test (b) Wilcoxon test (c) T test

Figure 7: Edge Map with delta=20 in Fig 2(b) (a) The proposed test (b) Wilcoxon test
5. CONCLUSION

The current paper proposed an efficient method for edge detection based on the Waerden rank statistic. Sobel and LoG operators with a threshold of 80 and Canny operator with a double threshold of 20 and 40 were used for edge detection in an original plane image and Gaussian noisy image.

Edge detection by the Wilcoxon and T test, as an example of a two-sample test, and the proposed Waerden test was also compared in an original plane image and Gaussian noisy image. The test statistics were computed based on modified gray levels obtained using an edge-height parameter to remove the noise and then compared with a threshold determined by the significance level \( \alpha = 0.0005 \) to determine the existence and nonexistence of an edge.

The experimental results confirmed that the proposed Waerden test was robust in both the noisy and noise-free images, while the Sobel, LoG, and Canny operators were all sensitive to the noisy image. In addition, the LoG method exhibited a tendency of detecting a partial edge where there was no edge. When the proposed Waerden test was directly compared with the Wilcoxon test, the proposed test produced better results at the same small significance.

REFERENCES